Paraconsistent Semantic Information

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One issue with Bar-Hillel and Carnap's account of semantic information is that it assigns maximal informativeness to contradictions, an issue that has been termed the *Bar-Hillel-Carnap Paradox*. What happens if we replace the underlying classical logic and probability with the paraconsistent LP (Logic of Paradox)? Does it resolve the Bar-Hillel-Carnap Paradox? Here is an investigation into the matter.

The paraconsistent logic LP is a many-valued logic with a possible worlds or possible states reading. This logic has three truth values; the classical t and f, representing 'true' and 'false' respectively, plus the truth value b, representing 'true or false'. Both t and b are designated value. Here are the negation, conjunction and disjunction connectives for this logic:

$f_\neg$	
t	f
b	b
f	t

$f_\wedge$	t	b	f
t	t	b	f
b	b	b	f
f	f	f	f

$f_\vee$	t	b	f
t	t	t	t
b	t	b	b
f	t	b	f

Now, take the following truth table:

A	B	$A \wedge \neg A$	$A \wedge B$	$(A \wedge \neg A) \vee (B \wedge \neg B)$
t	t	f	t	f
t	b	f	b	b
t	f	f	f	f
b	t	b	b	b
b	b	b	b	b
b	f	b	f	b
f	t	f	f	f
f	b	f	f	b
f	f	f	f	f

Your basic contradiction $A \wedge \neg A$ is still the most informative. Here $cont(A \wedge \neg A) = 1 - \frac{3}{9} = \frac{6}{9}$. The highest possible *cont* value for a classically satisfiable statement is $1 - \frac{4}{9} = \frac{5}{9}$. For example, the statement $A \wedge B$ has $cont = 1 - \frac{4}{9} = \frac{5}{9}$. There is no classically satisfiable statement that has a *cont* more than $\frac{5}{9}$ (true in 3 or less worlds). Here's why:

- If a formula is a classically satisfiable statement, then it is true in at least one classical possible world

- Each classical possible world has three non-classical possible worlds that correspond to it. If non-classical world N corresponds to classical world C, it means that no classically satisfiable statement can distinguish between C and N. Here are the correspondences:

Classical World		Non-Classical Correspondents	
A	B	A	B
t	t	t	t
		b	t
		b	b
t	f	t	b
		b	b
		b	f
f	t	f	b
		b	b
		b	t
f	f	f	b
		b	b
		b	f

- Everytime a classical statment is true in a classical world, it is also designated in the non-classical corresponding worlds. Firstly, we can convert any LP formula to conjunctive normal form. If a statement is classically true, then all of its conjuncts are true and in turn for every conjunct at least one of the disjuncts is true.

  This statment is also satisfiable in all the non-classical corresponding alternatives. With every true conjunct, get one of the disjuncts that is true. Say the disjunct is of the form p. Its classical valuation of $v(p) = t$ can be either:

  1. left alone to remain the same, if $v(p) = t$ in the non-classical alternative.
  2. replaced by $v(p) = b$, in which case p is still designated.

  If it is of the form $\neg p$, then its classical valuation of $v(p) = f$ can be either:

  1. left alone to remain the same, if $v(p) = f$ in the non-classical alternative.
  2. replaced by $v(p) = b$, in which case $\neg p$ is still designated.

However, some contradictions are less informative than some non-contradictions. For example, the statement $A \wedge B$ has *cont* $= 1 - \frac{4}{9} = \frac{5}{9}$, but the contradiction $(A \wedge \neg A) \vee (B \wedge \neg B)$ has *cont* $= 1 - \frac{5}{9} = \frac{4}{9}$. Can a formula of the form $\phi \wedge \neg\phi$ be found that has a *cont* less than $\frac{5}{9}$. Simply defining ϕ as $(A \wedge \neg A) \vee (B \wedge \neg B)$ gives us such a formula, so that $\mathit{cont}(((A \wedge \neg A) \vee (B \wedge \neg B)) \wedge \neg((A \wedge \neg A) \vee (B \wedge \neg B))) = \frac{4}{9}$

In order to get a classically satisfiable formula that is more informative than a basic contradiction, connective functional expressivity beyond that afforded by the $\neg$, $\wedge$ and $\vee$ of LP is required.

The logic RM_3 provides a conditional with the right type of functional expressivity. Its conditional is given by:

$f_\vee$	t	b	f
t	t	f	f
b	t	b	f
f	t	t	t

In RM_3, the formula $(A \equiv B) \wedge A$ is designated in less worlds than the formula $A \wedge \neg A$ is.

A	B	$A \to B$	$B \to A$	$(A \equiv B) \wedge A$
t	t	t	t	t
t	b	f	t	f
t	f	f	t	f
b	t	t	f	f
b	b	b	b	b
b	f	f	t	f
f	t	t	f	f
f	b	t	f	f
f	f	t	t	f