

Value Aggregate Truthlikeness and C-Monotonicity

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In two recent papers [1, 2] the relationship between (AGM) belief change and a particular approach to verisimilitude (truthlikeness), namely the ‘basic feature approach’ (BF-approach), is investigated. Basically, the BF-approach just deals with statements in conjunctive normal form, where each conjunct is of the form p or $\neg p$, with p being an atom.

Here are some of the relevant key points pertaining to the BF-approach:

- Takes a classical propositional logic framework with n atoms p_1, \dots, p_n . So there are 2^n possible states.
- The possible basic features are described using *literals*; either an atomic statement p_i or its negation, $\neg p_i$. A literal is denoted $\pm p_i$.
- A *constituent* or *state description* describes a specific possible state and has the form $\pm p_1 \wedge \dots \wedge \pm p_n$.
- There is one true constituent C_* , corresponding to the one true or actual state.
- This framework is described by propositional language L_n . A *conjunctive theory* (or c-theory) in L_n is a conjunction of k literals with k different atomic statements. They have the following form: $\pm p_1 \wedge \dots \wedge \pm p_k$, where $0 \leq k \leq n$
- A literal $\pm p_i$ occurring as a conjunct of a c-theory T is a basic claim (b-claim) of T . The set of all the b-claims of a c-theory is referred to as the basic content (b-content) of T .
- $t(T, C)$ is the true b-content of T with respect to constituent C (number of true b-claims in T relative to the state described by C), while $f(T, C)$ is the false b-content of T with respect to C .

In [2] the following comparative notion of verisimilitude for c-theories is given:

Definition Given two c-theories T_1 and T_2 , T_2 is *more verisimilar* than T_1 - in symbols, $T_2 >_{vs} T_1$ - iff at least one of the following two conditions holds:

$$(M_t) \quad t(T_2, C_*) \supset t(T_1, C_*) \text{ and } f(T_2, C_*) \subseteq f(T_1, C_*)$$

$$(M_f) \quad t(T_2, C_*) \supseteq t(T_1, C_*) \text{ and } f(T_2, C_*) \subset f(T_1, C_*)$$

Also, a verisimilitude measure is ‘conjunctively monotonic’ (c-monotonic) given the following definition:

Definition A verisimilitude measure V_s is *c-monotonic* just in case V_s satisfies the following condition:

C-monotonicity. Given two c-theories T_1 and T_2 , if $T_2 >_{vs} T_1$ then $V_s(T_2) > V_s(T_1)$.

As pointed out, a few known verisimilitude measures are not c-monotonic; Niiniluoto’s favoured ‘min-sum’ measure is one of them [2].

It turns out that my value aggregate measure [3] is also not c-monotonic:

Example Take a 9-proposition logical space, consisting of propositions $\{p_1 - p_9\}$. Let:

- $C_* = p_1 \wedge p_2 \wedge p_3 \wedge p_4 \wedge p_5 \wedge p_6 \wedge p_7 \wedge p_8 \wedge p_9$
- $T_1 = p_1$
- $A = \neg p_2$
- $T_2 = T_1 + A = p_1 \wedge \neg p_2$
- $t(T_1, C_*) = \{p_1\} \supseteq t(T_2, C_*) = \{p_1\}$
- $f(T_1, C_*) = \{\} \subset f(T_2, C_*) = \{\neg p_2\}$
- So $T_1 >_{Vs} T_2$
- $\text{info}()$ denotes the value aggregate equivalent of a verisimilitude measure
- $\text{info}(T_1) = 0.77 < \text{info}(T_2) = 0.79167$

I wonder what to make of this.

References

- [1] G. Cevolani and F. Calandra. Approaching the truth via belief change in propositional languages. In *EPSA Epistemology and Methodology of Science: Launch of the European Philosophy of Science Association*, pages 47–62, 2010.
- [2] G Cevolani, V Crupi, and R. Festa. Verisimilitude and belief change for conjunctive theories. *Erkenntnis*. Published online: 22 June 2011. URL = <http://www.springerlink.com/content/b1160810311u2x7n/>.
- [3] S. D’Alfonso. On quantifying semantic information. *Information*, 2(1):61–101, 2011. URL = <http://www.mdpi.com/2078-2489/2/1/61/>.