Truthlikeness and the Conjunction Fallacy

Simon D'Alfonso

The conjunction fallacy describes the judgement that a conjunction of two events is more probable than both of the individual events. The classic example of this fallacy is found in the work of Tversky and Kahneman [3], who presented the following fictitious scenario to subjects:

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

The subjects were asked which of the following two they thought more probable:

- 1. Linda is a bank teller.
- 2. Linda is a bank teller and is active in the feminist movement.

Amongst the subjects there was a tendency to judge 2 as more probable than 1; in fact, out of 142 university students who were simply asked to choose which they judged more probable, 85% of them chose 2.

This behaviour conflicts with the basic conjunction principle of probability theory, according to which the probability of a conjunction is always less than or equal to the probability of each conjunct individually: $\Pr(A \land B) \leq \Pr(A)$ and $\Pr(A \land B) \leq \Pr(B)$.

A range of responses to this puzzling phenomenon have been proposed. An interesting take on the paradox is presented in a recent paper titled 'The whole truth about Linda: probability, verisimilitude, and a paradox of conjunction' [1], in which the authors suggest estimated truth-likeness as an attribute guiding participants' prevailing responses. Estimated truthlikeness is "an independently motivated and formally definable epistemological notion relying on which many judges would rank 'feminist bank teller' over 'bank teller' in the Linda problem" [1, p. 612.].

So whilst the probability of the conjunction is always less than the probability of each of its conjuncts, the estimated truthlikeness of the conjunction can be higher than the estimated truthlikeness of one or both of its conjuncts. Here is a small formal demonstration of this idea using the average distance truthlikeness measure advocated by Tichy and Oddie.¹

Example Let b stand for 'Linda is a bank teller' and f stand for 'Linda is active in the feminist movement'. Say that the subject assigns different subjective probabilities to each of the four possibilities, as represented in Table 1, believing it most likely that Linda is both.

Given this we have:

¹For a description of this measure and its use in estimated truthlikeness calculations see [2].

State	b	f	Pr()
w_1	Т	Τ	$\frac{6}{16}$
w_2	Т	F	$\frac{4}{16}$
w_3	F	Τ	$\frac{4}{16}$
w_4	F	F	$\frac{2}{16}$

Table 1: Linda scenario

- $Pr(b) = Pr(f) = \frac{10}{16}$
- $\Pr(b \wedge f) = \frac{6}{16}$
- $\operatorname{Tr}_{est}(b|\top) = \operatorname{Tr}_{est}(f|\top) = \frac{9}{16} = 0.5625$
- $\operatorname{Tr}_{est}(b \wedge f | \top) = 0.625$
- $:: \operatorname{Tr}_{est}(b \wedge f | \top) > \operatorname{Tr}_{est}(b | \top) = \operatorname{Tr}_{est}(f | \top)$

References

- [1] Gustavo Cevolani, Vincenzo Crupi, and Roberto Festa. The whole truth about linda: probability, verisimilitude, and a paradox of conjunction. In Marcello D'Agostino et al., editor, *New Essays in Logic and Philosophy of Science*, pages 603–615. College Publications, 2010.
- [2] Simon D'Alfonso. On quantifying semantic information. *Information*, 2(1):61–101, 2011. URL = http://www.mdpi.com/2078-2489/2/1/61/>.
- [3] A. Tversky and D. Kahneman. Extensional versus intuitive reasoning: the conjunction fallacy in probability judgment. *Psychological Review*, 90(2):293–315, 1983.