The Logic Of Being Misinformed

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Abstract

It is well established that the states of knowledge and belief have been captured using systems of modal logic. Referred to respectively as epistemic and doxastic modal logics, they have been studied extensively in the literature. In a relatively recent paper entitled 'The Logic Of Being Informed' [8], Luciano Floridi does the same for the state of being informed, giving a logic of being informed also based on modal logic. In this *information logic* (**IL**), the statement $\mathrm{I}_a p$ stands for 'a is informed that p' or 'a holds the information that p'.[1] After a review of Floridi's logic of being informed, including an explication of the central concept of semantic information, I go on to develop a complementary logic of being misinformed, which formally captures the relation 'a is misinformed that p'.

1 Semantic Content and Information

Before taking a look at the logics in question, an explication of some fundamental concepts is in order. If σ is an instance of semantic content as understood here, then:

1. σ consists of one or more data
2. the data in σ are well-formed
3. the well-formed data in σ are meaningful

So data are the stuff of which semantic content is made; semantic content cannot be dataless, but, in the simplest case, it can consist of a single datum. A general definition of a datum is:

A datum is a putative fact regarding some difference or lack of uniformity within some context.[2]

[1] Of course, whether or not epistemic and doxastic logics adequately capture knowledge and belief is open to debate. But as Floridi points out, the task he sets himself is to determine an information logic, different from epistemic logic and doxastic logic, that formalises the relation "a is informed that p" equally well. The keyword here is "equally" not "well". He argues that **IL** can do for "being informed" what epistemic logic does for "knowing" and what doxastic logic does for "believing". If one objects to the last two, one may object to the first as well, yet one should not object to it more.

[2] See http://plato.stanford.edu/entries/information-semantic/#1.3 for discussion of this definition

Some examples will help to clarify the essence of this definition. Take a single sheet of unmarked white paper. It is an example of complete uniformity; each unit of the paper's surface is the same as every other unit.[3] As it is, there is no datum associated with this sheet. If a black marker were used to place a black dot on the sheet, then there would be a lack of uniformity. The white background plus the black dot would constitute the datum.[4]

Or as another example, consider a unary alphabet, consisting of the symbol 0. Any source that continuously emits symbols from this alphabet is not emitting data, for there is no lack of uniformity in its output. However, if the alphabet were expanded to include the symbol 1 as well as the symbol 0, then it would be possible for the source to emit data, by using both instances of the 0 symbol and instances of the 1 symbol.

With condition 2, 'well-formed' means that the data are composed according to the rules (syntax) governing the chosen system, code or language being analysed. Syntax here is to be understood generally, not just linguistically, as what determines the form, construction, composition or structuring of something. The string 'the an two green four cat !?down downx' is not well-formed in accordance with the rules of the English language, so therefore cannot be an instance of semantic content in the English language. Or, to take another example, the string '$A\neg B$' is not well-formed in accordance with the rules of the language of propositional logic, so therefore cannot be an instance of semantic content in propositional logic.

With condition 3, 'meaningful' means that the well-formed data must comply with the meanings (semantics) of the chosen system, code or language in question. For example, the well-formed string 'Colourless green ideas sleep furiously' cannot be semantic content in the English language because we may say (without getting into a debate about theories of meaning) that it is meaningless; it does not correspond to anything. Finally, an example of a string which fulfills conditions 1, 2 and 3 is 'The native grass grew nicely in spring'. Following are some cases of semantic content.

- A map of Europe contains the true semantic content that Germany is north of Italy, in the language of cartography. The data that this semantic content is made of is identified with the sheet of paper on which the map is printed plus the various markings on the page. This data is well-formed; among other things, the North-South-East-West coordinates are correctly positioned and no countries are marked as overlapping each other. Finally, this data is meaningful. Each part of the paper, contained in a thick black line and shaded in a certain colour corresponds or refers to a country. Thin blue lines mean rivers, etc.

- A person's nod contains the true semantic content that they are in agreement, in certain human body languages. The data that this semantic content is made of is indentified with the variation in head position. This data is well-formed; head movement is a legitimate expression in the language. This

[3]Whatever a unit might be measured in, pixels, millimetres, etc. Also, when comparing units, the attribution of sameness is based only on a certain property, namely that each unit is in its original state of unmarked whiteness. In certain ways each unit might differ. For example, each unit is in a different part of the sheet of paper, some units might be smoother than others. In this case, we are talking about the state of each unit in terms of its marking.

[4]This involves the notion of Taxonomic Neutrality. A datum is a relational entity. Neither of these two *relata*, the black dot or the white background, is the datum. Rather both, along with the fundamental relation of inequality between the dot and the background constitute the datum.

data is also meaningful; this particular expression means ‘yes’ or ‘positive’.

- The content of an Encyclopaedia Britannica entry on Italy will contain the true semantic content that Rome is the capital of Italy, in the language of English. The data that this semantic content is made of is indentified with the varied string of English alphabet symbols that constitute the entry. This data is well-formed as it accords with the syntax of the English language. It is also meaningful to an English language reader.

- The content of a book which says that there are nine planets in the solar system is false semantic content. The data that this semantic content is made are the varied strings of English alphabetical symbols. This data is well-formed as it accords with the syntax of the English and is also meaningful to an English language reader.

Truth and falsity supervene on semantic content, so semantic content can be either true or false. The General Definition of Information (GDI)[5] identifies information with semantic content. There are two main types of such *semantic* information, factual and instructional. Our interest here lies with factual semantic information, which is semantic content that is about some state of affairs, about some fact.[6]. Factual information comes in a variety of forms, as can be seen in the above cases. Ultimately though, these various forms of semantic information are reducible to propositional form, or propositional expression. If p is factual information, then it can be expressed in the form ‘the information that p’. This leads to an identification of information with propositions.[7]

Despite GDI, the alethic nature of information has been, and continues to be a point of contention. As will be further touched upon, some, including Floridi [8], have argued that for semantic content to be counted as information it must also be true. False information or *misinformation* (i.e. false semantic content) is not a type of genuine information. Without delving into this debate, a veridicality requirement of information is taken as a given in this paper; if something is information then it is true semantic content.

1.1 Another Requirement on Semantic Information?

By now it is well-established that according to the definition of information used in this paper, the following conditions must be met in order for σ to count as an instance of information:

1. σ consists of one or more data

2. the data in σ are well-formed

3. the well-formed data in σ are meaningful

[5]See [12]

[6]Unless otherwise specified, unqualified usage of the term ‘information’ throughout this paper refers to factual semantic information

[7]The reducibility of different kinds of semantic information to propositional form is, or at least I think it is, straightforward. For a discussion of this point, see [11, pg. 146]. Fox cogently argues that information should be understood in terms of propositions, what he calls the propositional analysis of information. [13, pg. 75]

[8]See [7], [8] and [9]

4. σ is true

These conditions are necessary. But are they also sufficient? I would like to now discuss the prospect of adding another condition to the definition of semantic information. This condition stems from certain conditions of information flow and its realisation was prompted by a recent point in the literature.

In [11], Floridi writes

> Epistemic luck does not affect informativeness negatively. To see why, one may use a classic Russellian example: if one checks a watch at time t and the watch is broken but stopped working exactly at $t-12$ hours and therefore happens to indicate the right time $t-12$ at t, one still holds the information that the time is t, although one can no longer be said to know the time. [11, pg. 144]

The point expressed in this quote seems counter to a certain prevalent view concerning the requirement of regularity for the presence of information flow. For example, in Knowledge and the Flow of Information, Dretske expresses a theoretical definition of a signal's (structure's) informational content in the following way:

A signal r carries the information that s is F = The conditional probability of s's being F, given r (and k), is 1 (but, given k alone, less than 1)

According to this definition, the watch is not giving information about the time, which implies that one cannot here be informed of the time with this watch. In fact with Dretske's type of information-theoretic epistemology, the requirement of regularity for information flow is supposed to explain why knowledge (defined as information-caused belief) is not present in the above example.

This general point has also been recognised by others:

> holding a piece of well-formed meaningful data which incidentally happens to be true is not sufficient for being informed. This we see by considering Gettier-like cases in which true consequences can be derived from a set of (partially) false data [2, p. 4].

How might one address the contrariety in this case between someone

1. holding the information that the time is t according to Floridi
2. not being informed that the time is t according to standard accounts of information and being informed

Firstly, if being informed that p is equivalent to holding the information that p as given in (1) and (2) above, then this inconsistency cannot be resolved. Also, one could reject (2) but it seems clear to me that this is not an acceptable option; the person is *not* informed of the time.

There is a sense in which holding the information that p is different to being informed that p; one might *hold* the information that p as an encrypted string which they do not have the resources to decrypt, thus

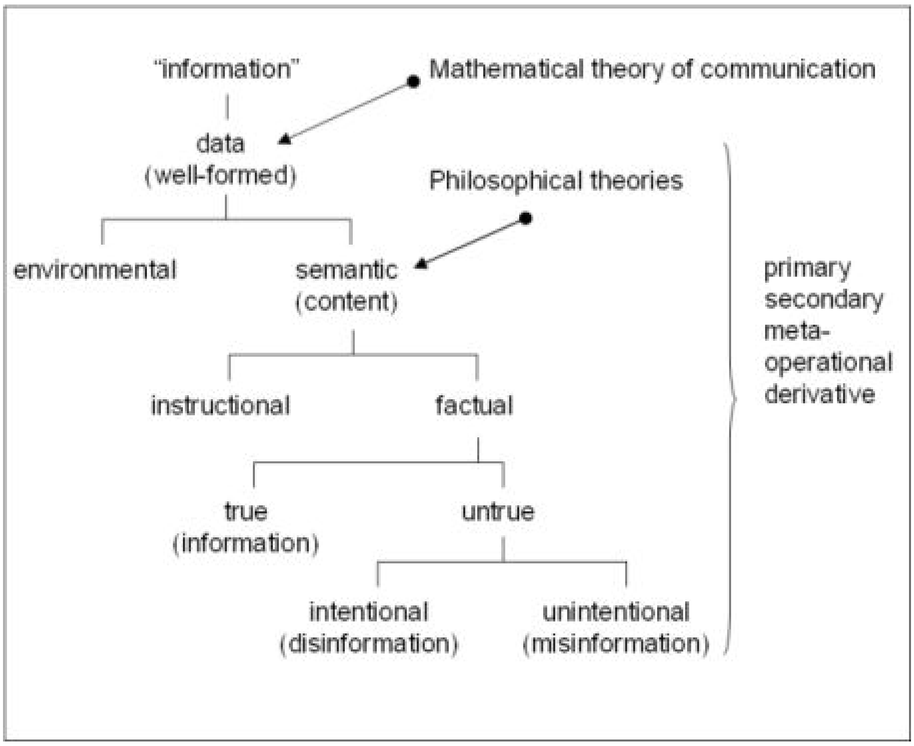


Figure 1: An informational map

they cannot access or be informed of this information (Floridi's logic of being informed captures this relation of holding information). In this case though, such an explanation could not apply, for the semantic content or information (the signal from the watch) can be accessed by the informee[9].

If one were to maintain both (1) and (2) here, another explanation must be given for why semantic information is just true semantic content but the state of being informed that p is more than just holding the semantic information that p.

The best option to me seems to reject (1); what the person holds is true semantic content but not information. In order to discuss this let us begin by establishing an important distinction between two types of information. Floridi [12] offers the informational map depicted in Figure 1.

With regards to this map, the type of information that Dretske is defining is environmental information. When a certain regularity exists between the signal of a clock and the actual time, the clock is providing environmental information about the time. On the other hand, when someone reads a book to find out the capital city of Finland, they come to hold the semantic information that Helsinki is the capital of Finland.

Despite this distinction, I contend that a link between the two types of information should be made. Environmental information precedes semantic information, which is true semantic content that is linked to environmental information. Environmental information is a result of regularities that exist within a distributed system [4, p. 7]. This environmental information is independent of any recipient, informee or semantic content bearer. If A carries the information that B, then if it is a fact that A it is a fact that B.

[9]Whilst the distinction between an agent merely holding a piece of information in this sense and actually being informed (able to access the information) is an interesting one, in this paper 'being informed that p' is treated as 'holding the information that p' in this sense of holding information.

Also, the reception of environmental information need not involve any semantics. For example, plants (e.g., a sunflower), animals (e.g., an amoeba) and mechanisms (e.g., a photocell) are certainly capable of making practical use of environmental information even in the absence of any (semantic processing of) meaningful data.[12]

Of course environmental information can be semantically processed as or into meaningful, well-formed data and thus transferred to a semantic content bearer. Take an example where the actual time is 6pm and someone is looking at a correctly functioning 24 hour format digital clock. They receive and process the signal coming from this clock. In doing so, the environmental information that it is 6pm, carried by the clock's signal of '18:00' is transferred to the semantic information that it is '18:00' which can be translated to the semantic information that it is 6pm. The semantic information '18:00' is meaningful, well-formed veridical data in the system of 24 hour time.

But if someone is looking at a broken clock that happens to be indicating the actual time of 6pm, then all they have obtained is the meaningful, well-formed veridical data that it is 6pm. Whilst it might seem straightforward and appropriate to suggest that information has also been obtained this is too problematic. Firstly, a tight connection between environmental and semantic information accords with intuitions and supports a bridge between information and knowledge. Secondly, accepting that this is a case of semantic information results in vulnerability to more extreme cases, where the unacceptability of treating the meaningful, well-formed veridical data as semantic information is clear to see.

The following examples increasingly illustrate this point. Firstly, imagine that a person you have just met informs you that they are from America. Based on the knowledge that California is America's most populated state, you guess and form the belief that this person is from California. It so happens that this person is from California. But should the meaningful, well-formed veridical data that constitutes the content of your belief count as information?

Secondly, take an even more problematic scenario where you are to guess the number of jelly beans in a jar. You take a random guess that there are 214 jelly beans in the jar and 214 happens to be the actual number of jelly beans in the jar. Here you have surely not been informed that there are 214 jelly beans in the jar and nor can you be said to have the information that there are 214 jelly beans in the jar.

These two examples involve true semantic content whose generation is not attributable to a sufficient source of information. In the first example, only partial information contributes to generation of the true semantic content that the person in question is from California. In the second example, the lucky guess is based on no information.

The only thing that differentiates the broken clock example from these two examples is that the source of semantic content generation (i.e. the visual signal of a clock being in a certain state) is a non-genuine 'replication' of the scenarios where there is actual information flow. But surely this is not the type of thing that should be the difference between true semantic content being information and true semantic content not being information. Also, unless a definition of semantic information requires origination from an information carrying source, we leave open the possibility of counting as information true semantic content that is generated from false semantic content.

All this suggests that a revisionary addition be made to the list of conditions for semantic information. If σ is an instance of semantic information:

1. σ consists of one or more data
2. the data in σ are well-formed
3. the well-formed data in σ are meaningful
4. σ is true
5. σ originates from an information carrying source

1.2 Misinformation

Whilst there is debate regarding the alethic nature of information, the alethic nature of misinformation is uncontentious:

> "'x misinforms y that p' entails that $\neg p$ but 'x informs y that p' does not entail that p [and since] we may be expected to be justified in extending many of our conclusions about 'inform' to conclusions about 'information' [it follows that] informing does not require truth, and information need not be true; but misinforming requires falsehood, and misinformation must be false." [13, pp. 160-1, 189, 193]

Put simply, misinformation is identified here with false semantic content; if an agent holds the misinformation that p or is misinformed that p, then p is false. A further qualification can be made between *misinformation* and *disinformation*, whereby the former is false or inaccurate information in general and the latter is false or inaccurate information that is spread intentionally. For the purposes of this paper, this difference is insignificant, as we are concerned with the core common idea of false or inaccurate information.

As touched upon earlier, although it is common or convenient to define misinformation as false information, misinformation is not a genuine type of information but rather pseudo-information; genuine semantic information requires truth. As Dretske puts it:

>]...] false information and mis-information are not kinds of information - any more than decoy ducks and rubber ducks are kinds of ducks.[5, p. 45.]

As Floridi points out [7, p. 364.], in some cases, 'false' is used *attributively* and in some cases it is used *predicatively*. The term 'false proposition' uses 'false' predicatively; a false proposition is a genuine proposition that is also false. On the other hand, the term 'false duck' uses 'false' attributively; when we say that something is a false duck we are in effect saying that it is not a duck. Likewise, with the term 'false information', 'false' is being used attributively.

Given all of this, we end up with the following definition of misinformation. If σ is an instance of misinformation, understood as semantic content:

1. σ consists of one or more data
2. the data in σ are well-formed
3. the well-formed data in σ are meaningful
4. σ is false

Unlike a definition of information, which arguably calls for an extra condition as discussed in Section 1.1, a definition of misinformation is less stringent and does not call for a further condition regarding the source of the misinformation.

2 The Logic Of Being Informed

We now take a look at Floridi's logic of being informed, both to get an idea about the enterprise and pave the wave for a logic of being misinformed. With the logic of being informed, the type of information being dealt with is factual information as semantic content, which can be expressed in the form 'the information that p'. Furthermore, the focus is on the statal condition into which an agent a enters, once a has acquired the information that p. When John consults a map of Europe to learn about the respective positions of Germany and Italy, he acquires the information that Germany is north of Italy. When Harry nods to John in response to John's question 'are you hungry Harry', John acquires the information that Harry is hungry. When John consults the Encyclopaedia Britannica entry on Italy to find out what its capital city is, he acquires the information that Rome is the capital of Italy.

The system of logic suggested by Floridi to formally capture the relation of 'being informed' is the normal modal logic system commonly referred to as **KTB** (also known as **B**, **Br** or **Brouwer's** system). So **IL** is constructed as an informational reading of **KTB**.

The modal operator corresponding to $\Box$ is interpreted as 'is informed that'. Replacing the symbol $\Box$ with the symbol I, and including a subscript reference to the agent involved, we get:

$$\mathrm{I}_a p\text{: } a \text{ is informed that } p$$

Floridi defines the accompanying modal operator corresponding to $\Diamond$ in the standard following way

$$\mathrm{U}_a p =_{def} \neg \mathrm{I}_a \neg p$$

which can be read as a is uninformed (is not informed, does not hold the information) that $\neg p$; or for all a's information (given a's information base), it is possible that p.

Furthermore:

a's information base can be modelled by representing it as a dynamic set D_a of sentences of a language L. The intended interpretation is that D_a consists of all the sentences, i.e. all the information, that a holds at time t. We then have that $I_a p$ means that $p \in D_a$, and $U_a p$ means that p can be uploaded in D_a while maintaining the consistency of D_a, that is, $U_a p$ means $\Diamond(p \in D_a)$.

It is important to note that Floridi's approach here is syntactical, rather than semantic in nature. Axioms are assessed for suitability based on how they accord with an informational reading. A semantic approach on the other hand would provide an interpretation of the Kripke-style semantics involved. For example, with the standard epistemic interpretation of Kripke-style semantics, a knows that p means that in all possible worlds compatible with what a knows, it is the case that p. No such account of informational accessibility is provided by Floridi.[10]

In arriving at his selection of the system **KTB**, Floridi considers 11 modal logic axiom schemata and systematically justifies his inclusion of some and exclusion of others. Here is a list of them, followed by some commentary:

Label	Definitions of Axiom Schemata	Name of the Axiom or Corresponding NML	Frame Property	Part of **IL**
A_1	$A \to (B \to A)$	1^{st} axiom of PC		√
A_2	$(A \to (B \to C)) \to ((A \to B) \to (A \to C))$	2^{nd} axiom of PC		√
A_3	$(\neg B \to \neg A) \to (A \to B)$	3^{rd} axiom of PC		√
A_4	$\Box A \to A$	KT or M, K2, veridicality	Reflexive	√
A_5	$\Box(A \to B) \to (\Box A \to \Box B)$	K, distribution, deductive cogency	Normal	√
A_6	$\Box A \to \Box\Box A$	4, S4, K3, KK, reflective thesis or positive introspection	Transitive	×
A_7	$A \to \Box\Diamond A$	KTB, B, Br, Brouwer's axiom or Platonic thesis	Symmetric	√
A_8	$\Diamond A \to \Box\Diamond A$	S5, reflective, Socratic thesis or negative introspection	Euclidean	×
A_9	$\Box A \to \Diamond A$	KD, D, consistency	Serial	√
A_{10}	$(\Box(A \to B) \to (\Box(B \to C) \to \Box(A \to C)))$	Single agent transmission		√
A_{11}	$\Box_x \Box_y A \to \Box_x A$	K4, multiagent transmission,or Hintikka's axiom		√

[10]Patrick Allo revisits and revises the logic of being informed in [1], where he discusses among other things an interesting informational interpretation of the accessibility relation.

2.1 IL Satisfies A_1, A_2, A_3, A_5

IL is trivially assumed to satisfy the axioms A_1 - A_3. As for axiom A_5, which gives the normal modal logic **K**, this is straightforwardly added, given that the relation of 'being informed' is distributive. One result of the system **K** which could be construed as problematic here concerns the closure of information, captured by the valid inference:

$$\mathrm{I}_a(p \supset q), \mathrm{I}_a p \vdash \mathrm{I}_a q$$

Fred Dretske has prominently argued against epistemic closure. In short, since the modes of gaining, preserving or extending knowledge, such as perception, testimony, proof, memory, indication, and *information* are not individually closed, neither is knowledge [15]. Despite this interesting argument, for now a basic **IL** is absolved from dealing with these more demanding notions.

2.2 Consistency and Truth: IL satisfies A_9 and A_4

A_4 simply represents the veridicality thesis, that for a proposition to be information it must be true. The inclusion of A_4 implies A_9. If an agent holds a piece of information p, then the agent cannot be informed of $\neg p$. Although it might be normatively desirable, I do not think that this is necessarily because the agent is consistent and cannot hold $\neg p$, but more so because $\neg p$ fails to qualify as information since it is false. It is possible for an agent to hold both the *semantic contents* or *data* p and $\neg p$, but since only one of them is true only one of them will count as information.

2.3 No reflectivity: IL does not satisfy A_6, A_8

The absence of the axioms A_6 and A_8 from this system of information logic sets it apart from standard epistemic logics. These axioms do not qualify for inclusion in a logic of being informed because informational agents need not be introspective. Whilst 'believing' and 'knowing' are *arguably* mental states that possess a reflective transparency, the relation of being informed does not require a mental or conscious state.[11] An artificial informational agent can possess the information that p without possessing the information that they posses this information. Even humans can be informed of something without being aware they have this information, the phenomenon of blindsight being a classic example. So $\mathrm{I}_a p \supset \mathrm{I}_a \mathrm{I}_a p$ does not hold. Similarly for $\mathrm{U}_a p \supset \mathrm{I}_a \mathrm{U}_a p$, which is equivalent to $\neg \mathrm{I}_a \neg p \supset \mathrm{I}_a \neg \mathrm{I}_a \neg p$; if an agent does not have the information that not-p, they need not be informed of their uninformed state.

2.4 Transmissibility: IL satisfies A_{10} and A_{11}

A_{10} is a theorem of all Normal Modal Logics. In the context of the logic of being informed, it simply captures the transitivity of information holding. A_{11} also makes perfect sense. If agent a holds the information that agent b holds the information that p, then a also holds the information that p. A derivation of this axiom

[11]'Arguably' is stressed here due to the debate over the KK principle, formalised in epistemic logic as: $\mathrm{K}p \supset \mathrm{KK}p$

requires usage of only a few core agreeable principles: (1) the veridicality of information (2) that agents are informed of the veridical nature of information (3) the distributivity of being informed.

$$\vdash \mathrm{I}_b p \supset p \quad (1)$$
$$\vdash \mathrm{I}_a(\mathrm{I}_b p \supset p) \quad (2)$$
$$\vdash \mathrm{I}_a(\mathrm{I}_b p \supset p) \supset (\mathrm{I}_a \mathrm{I}_b p \supset \mathrm{I}_a p) \quad (3)$$
$$\vdash \mathrm{I}_a \mathrm{I}_b p \supset \mathrm{I}_a p \quad (4)$$

2.5 Constructing the information base: IL satisfies A_7

A justification for the inclusion of A_7 in the context of information logic is not obvious, although some deliberation on the matter helps reveal its tentative plausibility. To start with, consider it in the form $p \supset \mathrm{I}_a \neg \mathrm{I}_a \neg p$. Floridi writes

> **IL** satisfies A_7 in the sense that, for any true p, the informational agent a not only cannot be informed that $\neg p$ (because of A_4), but now is also informed that a does not hold the information that $\neg p$. [8, p. 16]

As Floridi suggests, A_7 can be replaced with the axiom $\mathrm{U}_a \mathrm{I}_a p \supset p$. Floridi actually adopts $\mathrm{U}_a \mathrm{I}_a p \supset p$ instead of $p \supset \mathrm{I}_a \mathrm{U}_a p$ in his list of axioms for **IL**. Apparently "ontologically, this [axiom] is known to be a rather controversial result. Yet informationally, $\mathrm{U}_a \mathrm{I}_a p \supset p$ has a very intuitive reading" [8, p. 17]. This axiom seems to be saying something normative about the makeup of informational agents, that basically, agents are informed that they are not informed of false data. A good way to facilitate an attempt to interpret this axiom is by considering whether or not its negation should be satisfiable:

$\neg(\mathrm{U}_a \mathrm{I}_a p \supset p)$

$\neg(\neg \mathrm{I}_a \neg \mathrm{I}_a p \supset p)$

$\neg \mathrm{I}_a \neg \mathrm{I}_a p \wedge \neg p$

What would it mean for p to be false and $\neg \mathrm{I}_a \neg \mathrm{I}_a p$ to be true? Well firstly, if p is false, then an agent cannot hold the information that p. Since the possibility of an agent holding p as information is ruled out, then p cannot be in the agent's information base ($\neg \mathrm{I}_a p$). Secondly, agents have some type of meta-information regarding things that they cannot be informed of and since they are not and cannot be informed of p they have such meta-information about this.

Since $\neg \mathrm{I}_a \neg \mathrm{I}_a p \wedge \neg p$ affirms the first of these stipulations but not the second, it should not be satisfiable, therefore $\neg \mathrm{I}_a \neg \mathrm{I}_a p \supset p$ is valid. Despite this line of reasoning, the acceptability of this axiom is perhaps still somewhat debatable. See [1] for some more discussion on interpreting a symmetric informational accessibility relation.

Thus the normal modal logic which results, consists of the following axiom schemata and rules:

- $p \supset (q \supset p)$

- $(p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$
- $(\neg p \supset \neg q) \supset (q \supset p)$
- $\mathrm{I}p \supset p$
- $\mathrm{I}(p \supset q) \supset (\mathrm{I}p \supset \mathrm{I}q)$
- $\mathrm{UI}p \supset p$
- $\vdash p, \vdash p \supset q \Rightarrow \vdash q$ (Modus Ponens)
- $\vdash p \Rightarrow \vdash \mathrm{I}p$ (Rule of Necessitation)

Regarding the rules of the system, Modus Ponens is trivially accepted.

The Rule of Necessitation translates to the implication that an agent is informed about all theorems provable in the system. The epistemic logic equivalent of this phenomenon, what has been termed the 'logical omniscience' problem, has received its fair share of attention. Unless one is talking of an ideal epistemic agent, aware of all logical truths, it is problematic for an epistemic logical system which entails that an agent's knows all logical truths. Whilst it is thus very understandable to try and avert the logical omniscience issue, the information logic equivalent I would argue does not need to be averted with the same urgency, if at all, for it can be construed in an acceptable way.

In the cases of knowledge and belief, the fact that conscious mental states are required precludes the realistic possibility of logical omniscience. With information however, we are not talking about something which is present and accessible within a particular mental space of an agent. We are talking about the objective information that is contained within a system. Recalling certain distinctions between explicit belief/knowledge and implicit belief/knowledge [14], the notion of being informed here is aligned with the latter.

If an agent knows that p and knows that $p \supset q$, then they will only come to know that q once they have made that inference. On the other hand, if an agent has the information that p and the information that $p \supset q$, then they have the information that q, prior to and independent of the inference being made by them. So it is reasonable to say that in an informational agent equipped with the information of axioms and inference rules associated with a logical system, they are implicitly in possession of all the information that is entailed by the system.

A further supporting consideration is the fact that standard semantic accounts of information do not count tautologies as informative.

With Bar-Hillel and Carnap's account of semantic information [3], using some probability measure pr, two measures of information, $cont$ and inf are provided, such that:

$$cont(A) =_{df} 1 - pr(A)$$

and

$$inf(A) =_{df} -log_2(pr(A))$$

When the probability of A is 1, $cont(A) = 1 - 1 = 0$ and $inf(A) = -log_2(1) = 0$.

In another example, Fred Dretske's definition of informational content given in *Knowledge and the Flow of Information* [5] is

A signal r carries the information that s is F = The conditional probability of s's being F, given r (and k), is 1 (but given k alone, less than 1).

Here k stands for what the receiver already knows concerning the possibilities from the source. Once again, if the signal carries tautological content, then even without any signal and given k alone, the conditional probability would be 1. Since it would not be less than 1, the signal carries no information.

These accounts of information are based on the *Inverse Relationship Principle* [4], according to which the information carried by an event or structure is inversely proportional to its probability. Even an alternative approach to information such as Floridi's theory of strongly semantic information [6], which is based on truthlikeness measures, assigns an informativeness of 0 to tautologies.

The stipulation that tautological structures are not informative can be nicely accommodated by the idea that agents already possess such information by default, and since whatever information an agent already possesses can no longer be informative, tautologies are not informative.[12]

A weakening of the Rule of Necessitation could be in order though. The Weak Rule of Necessitation is as follows:

If p is a theorem of PC then Ip

So in replacing the standard Rule of Necessitation with its weak counterpart, it would still be the case that agents are informed of all tautologies. What would change is that being informed about tautologies would no longer be iterative, it would not follow that agents are informed of the fact that they are informed of a tautology.[13] An adoption of the Weak Rule of Necessitation could perhaps be based on the non-reflectivity of being informed, which was argued for in rejecting the positive introspection (A_6) and negative introspection (A_8) axioms. In [1] revisions to Floridi's logic of being informed result in a replacing of it with a non-normal modal logic equivalent, that uses the Weak Rule of Necessitation.

3 Logically Capturing Being Misinformed

What are some terms antonymous to knowing, believing and being informed? If one does not know that p, then they can be said to be *ignorant* of the fact that p. Epistemic logic provides a straightforward way to formalise ignorance of the fact that p as not knowing that p. Where K$_a p$ stands for 'a knows that p',

[12] There is a sense in which logical, mathematical and analytic truths can be informative, but this sense of informativeness differs to that being discussed here.

[13] The weak rule of necessitation is associated with non-normal modal logics.

$\neg K_a p$ stands for 'it is not the case that a knows p', or 'a is ignorant of the fact that p'. For belief, the term 'disbelief' is available. Where $B_a p$ stands for 'a believes that p', 'a disbelieves that p' can simply be represented with the statement $\neg B_a p$.

The state of being informed also has an opposite; in fact, it has two different types of opposites. The first and simplest opposite of being informed is being uninformed, where if one is not informed that p then they are uninformed that p. So the statement $\neg I_a p$ can be used to represent 'a is uninformed of that p'.[14]

The second type of opposite, namely *being misinformed*, differs in a radical way. Can being misinformed, like ignorance, disbelief and being uninformed, be defined in a similar way in terms of the logic of being informed?

In thinking about ways to logically capture the notion of being misinformed this way, two offhand candidates to represent 'a is misinformed that p' are $\neg I_a p$ and $I_a \neg p$.

Starting with the first of these options, we use $M_a p$ to stand for 'a is misinformed that p' and define it as $\neg I_a p$. With this approach the following formulas would be valid: $M_a p \supset \neg I_a p$ and $\neg I_a p \supset M_a p$. However, whilst the former of these formulas is right, the latter is clearly wrong; not being informed that p does not imply being misinformed that p.

Similarly in defining $M_a p$ as $I_a \neg p$, which would result in the valid formulas $M_a p \supset I_a \neg p$ and $I_a \neg p \supset M_a p$. Neither of these is acceptable. Regarding the former, being misinformed that p does not imply being informed that $\neg p$. Regarding the latter, being informed that $\neg p$ certainly does not imply being misinformed that p.

These negative outcomes prompt the development of a logic of being misinformed, where the main modal 'M' operator represents 'being misinformed' and is independent of a being informed operator 'I'. In the process, rather than first fully working out exactly what it means for an agent to be misinformed and then attempting to develop a logic that formally captures the resulting account, we will start off with our intuitions and basic ideas about misinformation and get a better, more refined idea about what it means for an agent to be misinformed of something as we go along.

3.1 A First Attempt

I begin by sketching a modal logic which *prima facie* is suitable for capturing the relation of being misinformed. The modal operator corresponding to $\Box$ will be represented using M, so that $M_a p$ stands for 'a is misinformed that p'. The modal operator corresponding to $\Diamond$ will be represented using $\langle M \rangle$, where $\langle M_a \rangle p$ stands for something like 'for all a's misinformation it is possible that p is false'.[15] We proceed by considering some of the important characteristic properties such a logic should have and surveying a range of modal axioms relative to the context. An obvious and first characterising axiom the logic should have is

$$M_a p \supset \neg p$$

[14] Or for another usage of the term 'uninformed', recall Floridi's usage in Section 2 for an operator corresponding to $\Diamond$.

[15] Note that throughout this paper the agent subscript will sometimes be omitted as a matter of convenience.

It captures the fundamental idea that if an agent is misinformed that p, then p must be false. This is simply the counterpart of the veridicality or **T** axiom of the logic of being informed, and can appropriately be called the falsity axiom. Adoption of this axiom ensures that $\mathrm{M}_a\mathrm{M}_a p \supset \mathrm{M}_a p$ does not hold; instead its corresponding falsity counterpart $\mathrm{M}_a\mathrm{M}_a p \supset \neg\mathrm{M}_a p$ does.

The next addition to the list is a counterpart to the so-called deontic axiom

$$\mathrm{M}_a p \supset \neg\mathrm{M}_a\neg p$$

This simply states that if one is misinformed that p, then it is not the case that they can be misinformed about not p. Since $\mathrm{M}_a p$ it follows that $\neg p$. Since $\neg p$ is true, then once cannot be misinformed that $\neg p$, because misinformation requires falsity. With the approaches taken here, adoption of the falsity axiom will *a fortiori* give this condition.

What other fundamental properties should be considered valid in the context of a logic of being misinformed? To aid further consideration, I note that replacing M_a with $\mathrm{I}_a\neg$ in the axiom $\mathrm{M}_a p \supset \neg p$ gives us $\mathrm{I}_a\neg p \supset \neg p$, which gives us $\mathrm{I}_a\neg\neg p \supset \neg\neg p$, which gives us $\mathrm{I}_a p \supset p$, its being informed counterpart. The inverse application of this process can aid further consideration in allowing for the exploration of what counterparts in a logic of being misinformed correspond to other axioms of the logic of being informed.

From the **K** axiom, $\mathrm{I}_a(p \supset q) \supset (\mathrm{I}_a p \supset \mathrm{I}_a q)$ we get $\mathrm{I}_a(\neg p \supset \neg q) \supset (\mathrm{I}_a\neg p \supset \mathrm{I}_a\neg q)$, which gives us $\mathrm{I}_a\neg(\neg p \wedge q) \supset (\mathrm{I}_a\neg p \supset \mathrm{I}_a\neg q)$, to get:

$$\mathrm{M}_a(\neg p \wedge q) \supset (\mathrm{M}_a p \supset \mathrm{M}_a q)$$

A justification of the validity of this counterpart to **K** is discussed shortly.

As for the other axioms of Floridi's logic of being informed, any corresponding transitive axiom is not to be included for pretty much the same reasons that Floridi rejected its inclusion in a logic for being informed; the state of being misinformed is not a reflective one. The same and beyond applies to rejection of any Euclidean axiom counterpart; whilst $\neg\mathrm{I}_a\neg p \supset \mathrm{I}_a\neg\mathrm{I}_a\neg p$ is justifiably rejected, it still retains some plausibility. However its misinformation counterpart with this process, $\neg\mathrm{M}_a p \supset \mathrm{M}_a\mathrm{M}_a p$, is plainly inappropriate.

Such a counterpart to the symmetric axiom, which is the defining axiom of the logic of being informed ($p \supset \mathrm{I}_a\neg\mathrm{I}_a\neg p$), finds no place either. It would translate to $p \supset \mathrm{M}_a\mathrm{M}_a p$, which clearly is not appropriate.

It is also noted that anything corresponding to the rule of necessitation (which results in an agent being informed of all valid formulas in **IL**) finds no place in a logic of being misinformed. $\vdash p \Rightarrow \vdash \mathrm{M}_a p$ is certainly not wanted, but neither is its counterpart $\vdash p \Rightarrow \vdash \mathrm{M}_a\neg p$; it certainly does not follow that agents are misinformed of all contradictions. Instead, an inference that we will want valid is $p \vdash \neg\mathrm{M}_a p$; if p is true then p is not misinformation, so an agent cannot be misinformed that p. An easy way to overcome this issue is by making this modal logic non-normal.[16] Though somewhat crude, it will serve for now as we focus on other key aims.

[16]This strategy has its origins in attempts to overcome the logical omniscience problem. Rantala [17] proposed to use non-normal modal logics based on impossible worlds to overcome the logical omniscience problem in epistemic and doxastic logics.

Guided by the above considerations and with this modest collection of desirable valid formulas we can start shaping a logic.

3.2 Semantics

Using the machinery of modal logic, this approach is based on inverting the modal operators in the following way. An interpretation for this logic is a structure $\langle W, N, R, v \rangle$ such that:

- W is a non-empty set of objects (possible worlds).
- $N \subseteq W$ is a set of normal worlds. $W - N$ gives the set of non-normal worlds.
- R is the binary accessibility relation on W, so that $R \subseteq W \times W$. Where $w_1 \in W$ and $w_2 \in W$, $w_1 R w_2$ means that world w_1 accesses w_2.
- v is the valuation function that assigns a truth value of 1 or 0 to each pair consisting of a world and proposition

The one constraint on R is that of reflexivity: for all w, wRw.

The conditions for the connectives $\neg$, $\wedge$ and $\vee$ are as follows: For any world $w \in W$:

- if $v_w(p) = 0$ then $v_w(\neg p) = 1$
- if $v_w(p) = 1$ then $v_w(\neg p) = 0$
- if $v_w(p) = 1$ and $v_w(q) = 1$ then $v_w(p \wedge q) = 1$
- if $v_w(p) = 0$ or $v_w(q) = 0$ then $v_w(p \wedge q) = 0$
- if $v_w(p) = 1$ or $v_w(q) = 1$ then $v_w(p \vee q) = 1$
- if $v_w(p) = 0$ and $v_w(q) = 0$ then $v_w(p \vee q) = 0$

For any normal world $w \in N$:

- if, for some $w' \in W$ such that wRw', $v_{w'}(p) = 0$, $v_w(\langle \mathrm{M} \rangle p) = 1$
- if, for no $w' \in W$ such that wRw', $v_{w'}(p) = 0$, $v_w(\langle \mathrm{M} \rangle p) = 0$
- if, for all $w' \in W$ such that wRw', $v_{w'}(p) = 0$, $v_w(\mathrm{M}p) = 1$
- if, for some $w' \in W$ such that wRw', $v_{w'}(p) = 1$, $v_w(\mathrm{M}p) = 0$

For any non-normal world $w \in W - N$:

- $v_w(\mathrm{M}p) = 0$
- $v_w(\langle \mathrm{M} \rangle p) = 1$

It can easily be verified that $\mathrm{M}p \equiv \neg\langle\mathrm{M}\rangle\neg p$.

Logical validity is defined in terms of truth preservation at all worlds, thus:

$\Sigma \models p$ iff for all interpretations $\langle W, N, R, v\rangle$ and for all $w \in W$: if $v_w(q) = 1$ for all $q \in \Sigma$ then $v_w(p) = 1$.

Aside from the inversion of the modal operators, the resulting system is akin to the non-normal system of modal logic $E2$[17]

$\mathrm{M}_a p$ is given a semantic interpretaion along the following lines:

$\mathrm{M}_a p$ - in all possible worlds compatible with a's misinformation, it is the case that $\neg p$

So if a is misinformed that $p \wedge q$, then for each of a's misinformational alternatives w, one of the following is the case

1. $v_w p = 0$ and $v_w q = 0$
2. $v_w p = 0$ and $v_w q = 1$
3. $v_w p = 1$ and $v_w q = 0$

Finally, non-normal worlds can be interpreted as states where an agent is not misinformed of anything.

3.3 Tableaux Proof System

Here is a simple tableaux proof system for this logic, with only select rules being detailed. For full details on the remaining tableaux rules consult [16], on which the tableaux style of this system is based. The non-modal rules are those for classical propositional logic.

Rules for the modal operators are:

$$\begin{array}{cc} \neg\mathrm{M}p, i & \neg\langle\mathrm{M}\rangle p, i \\ | & | \\ \langle\mathrm{M}\rangle\neg p, i & \mathrm{M}\neg p, i \end{array}$$

$$\begin{array}{cc} \mathrm{M}p, i & \langle\mathrm{M}\rangle p, i \\ iRj & | \\ | & iRj \\ \neg p, j & \neg p, j \end{array}$$

The rule for reflexivity is:

$$\begin{array}{c} . \\ | \\ iRi \end{array}$$

If world i occurs on a branch of a tableaux, call it M-inhabited if there is some node of the form $\mathrm{M}q, i$ on the branch. The rule for $\langle\mathrm{M}\rangle p, i$ is activated only when i is M-inhabited.

[17]see http://home.utah.edu/~nahaj/logic/structures/systems/e2.html

3.4 Some Results

So how do the valid and invalid inferences in this system accord with our picture of being misinformed thus far? To begin with we have the following three expected results:

1) $\vdash \mathrm{M}p \supset \neg p$

$$\begin{array}{c} \neg(\mathrm{M}p \supset \neg p), w_1 \\ | \\ \mathrm{M}p, w_1 \\ \neg\neg p, w_1 \\ | \\ p, w_1 \\ | \\ w_1 R w_1 \\ \neg p, w_1 \\ | \\ \times \end{array}$$

2) $\vdash \mathrm{M}p \supset \langle\mathrm{M}\rangle p$

$$\begin{array}{c} \neg(\mathrm{M}p \supset \langle\mathrm{M}\rangle p), w_1 \\ | \\ \mathrm{M}p, w_1 \\ \neg\langle\mathrm{M}\rangle p, w_1 \\ | \\ w_1 R w_1 \\ \mathrm{M}\neg p, w_1 \\ | \\ \neg p, w_1 \\ \neg\neg p, w_1 \\ | \\ \times \end{array}$$

3) $\vdash \neg\mathrm{M}\top$

$$\begin{array}{c} \neg(\neg\mathrm{M}\top), w_1 \\ | \\ \mathrm{M}\top, w_1 \\ | \\ \neg\top, w_1 \\ | \\ \times \end{array}$$

The counterpart to the **K** axiom of the logic of being informed is valid in this system; if an agent is misinformed that $\neg p \wedge q$, then if they are misinformed that p then they are misinformed that q.

$\vdash \mathrm{M}(\neg p \wedge q) \supset (\mathrm{M}p \supset \mathrm{M}q)$

$$\neg(\mathrm{M}(\neg p \wedge q) \supset (\mathrm{M}p \supset \mathrm{M}q)),\ w_1$$

$$\mid$$

$$\mathrm{M}(\neg p \wedge q), w_1$$

$$\neg(\mathrm{M}p \supset \mathrm{M}q), w_1$$

$$\mid$$

$$\mathrm{M}p, w_1$$

$$\neg\mathrm{M}q, w_1$$

$$\mid$$

$$\langle \mathrm{M} \rangle \neg q, w_1$$

$$\mid$$

$$w_1 R w_2$$

$$\neg\neg q, w_2$$

$$q, w_2$$

$$\mid$$

$$\neg(\neg p \wedge q), w_2$$

$$p \vee \neg q, w_2$$

$$\begin{array}{cc} p, w_2 & \neg q, w_2 \\ \mid & \mid \\ \neg p, w_2 & \times \\ \mid & \\ \times & \end{array}$$

This is a good result to continue with, as an explanation or reading of this result will help to shed some light on how misinformation might work in agents. Arguably, misinformation is not as amenable as information when it comes to a formal logical analysis and semantic interpretation. In order to proceed, a crucial idea must first be established. If an agent is misinformed that $p \wedge q$, then the agent also holds the semantic content (i.e. meaningful, well-formed data) that $p \wedge q$. From this, we can say that the agent also holds the semantic content that p and the semantic content that q. The legitimacy of this point, including what it means for an agent to hold some piece of semantic content, will be discussed in detail shortly. For now it can be accepted as a plausible and reasonable principle.

Going back to the logic of being informed, if an agent holds the information that $p \wedge q$, then they hold the information that p and the information that q. With misinformation, if an agent has the misinformation (i.e. false semantic content) that $p \wedge q$, then they hold the true/false semantic content that p and the true/false semantic content that q. For if the agent has the misinformation that $p \wedge q$, then it is actually the case that $\neg p \vee \neg q$, from which there are three possibilities:

- both the semantic content that p and the semantic content that q are false semantic content
- p is false semantic content and q is true semantic content
- p is true semantic content and q is false semantic content

	Formula	Wanted?	Valid?		Formula	Wanted?	Valid?
1	$(\mathrm{M}p \vee \mathrm{M}q) \supset \mathrm{M}(p \vee q)$	no	$\times$	7	$(\mathrm{M}p \wedge \mathrm{M}q) \supset \mathrm{M}(p \wedge q)$	yes	$\surd$
2	$(\mathrm{M}p \vee \mathrm{M}q) \supset \mathrm{M}(p \wedge q)$	no	$\surd$	8	$(\mathrm{M}p \wedge \mathrm{M}q) \supset \mathrm{M}(p \vee q)$	yes	$\surd$
3	$(\mathrm{M}p \vee \mathrm{M}q) \supset (\mathrm{M}p \wedge \mathrm{M}q)$	no	$\times$	9	$(\mathrm{M}p \wedge \mathrm{M}q) \supset (\mathrm{M}p \vee \mathrm{M}q)$	yes	$\surd$
4	$\mathrm{M}(p \vee q) \supset (\mathrm{M}p \vee \mathrm{M}q)$	no	$\surd$	10	$\mathrm{M}(p \wedge q) \supset (\mathrm{M}p \wedge \mathrm{M}q)$	no	$\times$
5	$\mathrm{M}(p \vee q) \supset (\mathrm{M}p \wedge \mathrm{M}q)$	no	$\surd$	11	$\mathrm{M}(p \wedge q) \supset (\mathrm{M}p \vee \mathrm{M}q)$	yes	$\times$
6	$\mathrm{M}(p \vee q) \supset \mathrm{M}(p \wedge q)$	no	$\surd$	12	$\mathrm{M}(p \wedge q) \supset \mathrm{M}(p \vee q)$	no	$\times$

Table 1: Summary of results

In this case, the agent has the misinformation that $\neg p \wedge q$, so the agent holds the semantic content that $\neg p$ and the semantic content that q. Now, since the agent is misinformed that $\neg p \wedge q$, it is actually the case that $p \vee \neg q$. If the agent holds the misinformation that p, then it is actually the case that $\neg p$. Since $\neg p$, it follows that $\neg q$. Finally, since the agent holds the semantic content that q when in fact $\neg q$, then the agent is misinformed that q.

So far so good. Unfortunately however, there are some particularly problematic results, problematic to the extent that they render this approach to logically capturing the relation of being misinformed unworkable overall. Table 1 is a summary of some other results in the system, from which I will choose a couple in particular to make this point. The 'Wanted' column indicates whether or not the formula in question is judged to be acceptable as a principle of being misinformed. The 'Valid' column indicates whether or not the formula in question is actually valid within this system. As can be seen, we have some valid results that we do not want and some invalid results that we do want.

Let us begin with the one example of the latter, Result 11. The following reasoning justifies why this formula is wanted. Firstly, if an agent is misinformed that $p \wedge q$, then $\neg p \vee \neg q$. Secondly, if an agent is misinformed that $p \wedge q$ then they hold the semantic content that $p \wedge q$ and so hold the semantic content that p and the semantic content that q. If it is the case that $\neg p$, then since the agent holds the semantic content that p then they are misinformed that p. Or if it is the case that $\neg q$, then since the agent holds the semantic content that q, then they are misinformed that q. So at least one of the disjuncts in $\mathrm{M}p \vee \mathrm{M}q$ hold.

The next result to scrutinise is Result 5. Whilst this inference is itself problematic, to make things simpler we shall focus on a consequence of it, the problematic validity of:

$$\vdash \mathrm{M}(p \vee q) \supset \mathrm{M}p$$

If an agent is misinformed that $p \vee q$, are they misinformed that p? Well, let p stand for 'Berlin is the capital of Italy' and let q stand for 'Paris is the capital of Italy.' If an agent is misinformed that Berlin is the capital of Italy or Paris is the capital of Italy, it seems wrong to say that they are misinformed that Berlin is the capital of Italy. Since both atomic propositions involved here are false, if an agent holds the semantic content of their disjunction, then that agent is rightly misinformed that $p \vee q$. But the holding of this misinformative disjunction does not imply the holding of any particular false disjunct.

Unlike with the case of $\vdash \mathrm{M}(\neg p \wedge q) \supset (\mathrm{M}p \supset \mathrm{M}q)$, this counter example cannot be plausibly explained away by appealing to the idea that an agent having misinformation implies that they also have the semantic content on which that misinformation is based. In the following discussion we are going to get clearer on the notion of an agent holding semantic content and the nature of semantic content within agents.

Since the agent is misinformed that $p \vee q$ it follows that $\neg p \wedge \neg q$. If it also follows that the agent has the semantic content that p, then it follows that they are misinformed that p. But there is no obvious, natural way to infer that an agent holds the semantic content that p just because they are misinformed that $p \vee q$ and hold the semantic content that $p \vee q$. But what does semantic content exactly mean here? What does this sense of holding semantic content entail, what are some of the agent's commitments or assumptions in holding a piece semantic content? As will become evident, the possession of or holding of semantic content by an agent at least implies that the agent has not rejected the truth or accepted the falsity of the semantic content. Further still, the retention of semantic content by an agent could imply that the agent presumes its truth. But bear in mind that there are cases in which an agent will not be aware that it holds a certain piece of semantic content and thus is not in a position to make a reflective assessment of it.

Now, if an agent is misinformed that p, then they have the semantic content that p, they have some data in their system, that is well-formed and meaningful and stands for the proposition expressed by p. So the representation of that semantic content within the agent's system requires a physical representation, a datum that p. This datum is present within the system to represent the semantic content that p. In a human agent this data would be instantiated as some brain structure perhaps. In a computer it could be instantiated as digitally encoded data stored on a hard disk. But this semantic content that p is more than just data about p. It is an individual piece of propositional content that the agent holds which is intended to correspond to a presumed fact. The following example will help illustrate this point. An agent can have a representation of $A \vee B$ in their database and a representation of $\neg(\neg A \wedge \neg B)$ in their database. Whilst these are distinct pieces of instantiated data, they are the same semantic content.

If an agent is misinformed that $p \vee q$, then they have the semantic content that $p \vee q$. If an agent has the semantic content that $p \vee q$, then both constituents of this disjunction require physical representations, so the agent will hold the datum that p and the datum that q. But these data are present as constituents in the representation of the semantic content that $p \vee q$, not for the representation of the standalone semantic content that p or the standalone semantic content that q. Although the semantic content $p \vee q$ is present within the system and is intended to correspond to the presumed fact that $p \vee q$, this does not imply that p will exist as an independent piece of semantic content intended to correspond to the presumed fact that p, for the presumed truth of p does not follow from the presumed truth of $p \vee q$.

Some ideas related to this discussion are touched upon in [2], where Patrick Allo elaborates a combined logic for data and information with the motivation to investigate formalisation of the 'No Information without Data-representation' principle. He distinguishes between two types of data holding relations, D'$_a A$ and D$_a A$. The former refers to the syntactic relation of holding a piece of data A. In this sense, $A \wedge B$ and $A \wedge B$ are the same data but $A \wedge B$ and $\neg(\neg A \vee \neg B)$ are different data. Thus "we get D'$_a[A; B]$ iff D'$_a A$ and D'$_a B$ as a minimal *logical* constraint on holding data (where square braces and semicolons are used to represent the complex syntactical objects obtained by concatenating simpler objects)" [2, p. 7.]. The latter refers to the semantic relation of holding data: "D$_a A$ expresses the semantic relation of holding data which carries the content that A as opposed to the syntactic relation of holding a piece of data A. As such, it does not necessarily refer to a particular object, but only to a state that is warranted by a particular object"[2, p. 9.]. It is this latter notion of data with which the notion of semantic content I use is aligned.

Any stipulation that an agent would come to have the standalone semantic content p in virtue of it having the semantic content $p \vee q$ is implausible. There is no realistic potential for an agent to generate the semantic content that p from the semantic content that $p \vee q$; it is not how human agents operate and any database that implemented such a system would be at odds with norms of reasoning and deduction.

The general problem with this first attempt at a logic for being misinformed is an insightful one. Take a falsity-preserving consequence relation $\vdash_F$, which is such that if $p \vdash_F q$, then every time p is false q is false. This system of misinformation logic is such that

$$p \vdash_F q \Rightarrow \mathrm{M}_a p \vdash \mathrm{M}_a q$$

which is why we get the problematic Result 5 above, since $p \vee q \models_F p$.

A logic for being informed on the other hand has a normal modal operator, so that

$$p \vdash q \Rightarrow \mathrm{I}_a p \vdash \mathrm{I}_a q$$

From these considerations a particularly pertinent point can be drawn. It is evident that information and truth preservation go hand in hand. If every time p is true then so is q, then if an agent is informed that p then they are also informed that q. On the other hand, misinformation and falsity preservation do not go hand in hand. If every time p is false then so is q, it does not follow that if an agent is misinformed that p then they are also misinformed that q. This discussion sheds some light on developing a system to capture how agents work with semantic content, something to which we shall now turn.

3.5 The Logic of Holding Semantic Content

The analysis in the previous section suggests that another way to approach a logic which formalises the relation 'agent a is misinformed that p' is to first get a logic which formalises the relation 'agent a holds the semantic content that p' and then define being misinformed in terms of holding false semantic content. This is the task to which we shall now turn. The intention here is not to argue for a definitive logic for holding semantic content but rather adopt a logic that plausibly captures this notion.

The operator 'S' will be used to correspond to $\Box$ and '$\langle\mathrm{S}\rangle$' to correspond to $\Diamond$. A straightforward way to get such a logic is by using a simple modal logic, though as far as modal logics go, such a logic would have relatively few constraining axioms. The base normal modal logic **K**, with its axiom $\mathrm{S}_a(p \supset q) \supset (\mathrm{S}p_a \supset \mathrm{S}_a q)$, is a good starting point. Beyond this, there is need for not much, if anything else.

The types of constraining axioms associated with epistemic, doxastic and informational modal logics find no place in a modal logic for semantic content. The **T** axiom ($\mathrm{S}_a p \supset p$) is certainly not wanted, since semantic content need not be true. Nor is the **D** axiom ($\mathrm{S}_a p \supset \langle \mathrm{S}_a \rangle p$), for an agent's semantic content base need not be consistent. A distinguished type of example for this would be that of an agent which can accommodate contradictory semantic content and persist in holding the semantic contents p and $\neg p$ until one of them is determined to be false. But this inconsistency tolerance need not be, and is not, a foundation for the type of agent in mind here.

More moderate and general types of examples in mind would involve an agent holding the semantic content that p and the semantic content that $\neg p$ whilst not being informed that they are holding contradictory semantic content. For example, an agent might hold the semantic contents that p and that $\neg p$, whilst not being informed that they hold one, the other or both. Put formally, whilst it could be the case that $\mathrm{S}_a p$ and $\mathrm{S}_a \neg p$, this does not imply something like $\mathrm{S}_a \mathrm{S}_a p \wedge \mathrm{S}_a \mathrm{S}_a \neg p$ or $\mathrm{I}_a \mathrm{S}_a p \wedge \mathrm{I}_a \mathrm{S}_a \neg p$. For that matter, nor does it even imply something like $\mathrm{S}_a \mathrm{S}_a p \vee \mathrm{S}_a S_a \neg p$ or $\mathrm{I}_a S_a p \vee \mathrm{I}_a S_a \neg p$. So in these cases one can hold semantic content without, for some reason, being aware that they hold that semantic content. It could be because they have not fully processed all the relevant semantic content. For example, let f stand for 'France is the most populated country in Europe' and let g stand for 'Germany is the most populated country in Europe'. Someone could be told (and misinformed) that France is the most populated country in Europe (so f and $\neg g$). They could also be told that the most populated country in Europe (actually Germany) shares borders with more European countries than any other country on the continent. If this person thought really hard about a map of Europe they had studied and stored in their memory, they would deduce that this country is Germany, and hence come to be aware that they hold contradictory semantic content (g and $\neg g$). But until this process occurs and one of the conflicting items of semantic content is discarded, they can be said to hold both the semantic content that g and the semantic content that $\neg g$.

Or for another type of example, one might simply not have introspective access to some particular semantic content because of its subconscious nature. Even if an agent held the semantic contents that p and that $\neg p$ and was aware of this, they might not be aware of a conflict between the two. Fregean and Kripkean puzzles come to mind here.

These points lead to rejection of the **4** axiom ($\mathrm{S}_a p \supset \mathrm{S}_a \mathrm{S}_a p$). Support for this rejection can be found in [2], where one of the core principles is that if an agent holds the information that p then they hold the data p ($\mathrm{I}_a p \supset \mathrm{D}_a p$). One of the tasks in this paper is an investigation into meta-data, which

> play a role that is similar to that of reflective states in standard epistemic logic. When B expresses the same proposition as $\mathrm{D}_a A$, then B is meta-data for A: meta-data about the data one holds. Trivially, $\mathrm{D}_a B$ then expresses that one holds such meta-data; in short: $\mathrm{D}_a \mathrm{D}_a A$. With this in mind, one should then ask how $\mathrm{D}_a \mathrm{D}_a A$ relates to $\mathrm{D}_a A$.[2, p. 10.]

The first principle to question is whether one cannot hold data without also having meta-data for it:

$$\mathrm{D}_a A \supset \mathrm{D}_a \mathrm{D}_a A \qquad (\mathbf{DD1})$$

The question regarding this principle can be put thus: should an agent be able to obtain the meta-data for all the data it holds by purely logical means? Allo calls this principle the 'free meta-data' thesis. He cogently argues that metadata should not come for free, one should not be able to obtain meta-data by logical means alone; hence the validity of DD1 is rejected.

Since semantic content is just well-formed, meaningful data, the rejection of the 'free meta-data' thesis can be extended to a rejection of $\mathrm{S}_a A \supset \mathrm{S}_a \mathrm{S}_a A$.

The second principle to question is its converse, whether there is no meta-data without data:

$$\mathrm{D}_a \mathrm{D}_a A \supset \mathrm{D}_a A \qquad (\mathbf{DD2})$$

Allo calls this the 'non-corrupted meta-data' thesis. This is a more plausible principle than its counterpart; if an agent has meta-data about some data A, then they hold the data A. Allo himself does not reject DD2 and it is normatively desirable. Hence $\mathrm{S}_a \mathrm{S}_a A \supset \mathrm{S}_a A$ is an acceptable principle.

Thus far we have a good idea on the nature of a modal logic that formalises the relation of holding semantic content. Without further discussion and analysis of the matter, we can say that for the purpose at hand the modal logic $\mathbf{K}$ plus the axiom $\mathrm{SS}p \supset \mathrm{S}p$ will suffice for such a logic.

Now that we have this basic logic of holding semantic content, an important justification for its most basic of valid inferences is to be established. The basic idea is that the generational nature of semantic content within an agent and the types of inferences an agent can or will make with the semantic content they possess are in line with the nature of and inferences they make with information. The type of inferences an agent can or will make regarding semantic content it possesses are inferences based on the assumption that the content is true; "As a rule, agents assume that some content is by default an instance of information"[10].

If an agent is misinformed that $p \vee q$, then $\neg p \wedge \neg q$. But as we have seen, although the agent holds the false semantic content that $p \vee q$ and it is the case that both p and q are false, this does not imply that the agent holds the false semantic content (i.e. is misinformed) that p (or q).

But consider what could be called the information counterpart of misinformation's $p \vee q$. Why is it the case that the inference

$$\mathrm{I}_a(p \wedge q) \vdash \mathrm{I}_a p \wedge \mathrm{I}_a q \qquad (\mathbf{I1})$$

is straightforwardly accepted? If $\mathrm{I}_a(p \wedge q)$ is true then p is true and q is true. But in order to secure $\mathrm{I}_a p$ and $\mathrm{I}_a q$, as well as p and q being true, the agent in question must also hold the semantic content p and the semantic content q. This process is implicit in the validity of I1. If an agent holds the information that $p \wedge q$, they implicitly hold the semantic content that p and the semantic content that q. For even if they do not explicitly hold the semantic content that p and the semantic content that q as physical data instantiations, the fact that they hold the information that $p \wedge q$ is enough for them to be able to generate the information and semantic content that p and the information and semantic content that q.

Thus the relation of holding semantic content is such that if $p \vdash q$, then $\mathrm{S}_a p \vdash \mathrm{S}_a q$. So if an agent holds the true semantic content (i.e. information) that $p \wedge q$ then they can be said to hold the true semantic content that p and the true semantic content that q. But if an agent holds the false semantic content (i.e. misinformation) that $p \vee q$, then although it is the case that $\neg p$ is true and $\neg q$ is true, they do not hold the semantic content that $\neg p$ (or p) nor the semantic content that $\neg q$ (or q), for these do not follow from the truth of $p \vee q$.

Also, if an agent were to make a valid inference based on the known falsity of $p \vee q$ and infer $\neg p \wedge \neg q$, then they would no longer be misinformed that $p \vee q$, it would no longer be misinformation in their system. Also, since they infer and it is actually the case that $\neg p$ and $\neg q$, and since they hold the semantic content that $\neg p$ and the semantic content that $\neg q$ rather than p and q, they cannot be misinformed that p or misinformed that q.

[***Still need to think about an interpretation of S accessibility relation. Any thoughts?***]

3.6 The Logic of Being Misinformed

We define being misinformed in terms of the logic of holding semantic content, such that

$$\mathrm{M}_a p =_d \mathrm{S}_a p \wedge \neg p$$

Unlike the state of being informed, which is taken to be prime and captured directly with its own operator, the state of being misinformed is defined in terms of the state of holding semantic content. An incidental consideration that supports this approach is a certain contrast between information and misinformation; as discussed earlier in Section 1.1, information is considered more than just true semantic content. As a consequence of this, "it is convenient to treat the state of being informed as a prime" [2, p. 4.]. Unlike the more sophisticated state of being informed, all the state of being misinformed requires is false semantic content, irrespective of the nature of its origins.[18]

One other thing worth mentioning here is that this notion of being misinformed captures a somewhat broader range of cases than the typical types of cases one might associate with being misinformed. The main sense in which a is misinformed that p is when $\mathrm{B}_a p \wedge \neg p$; John asks Bob 'what is the capital of Finland', Bob replies 'Oslo' which John accepts and John comes to have the false belief that Oslo is the capital of Finland, thereby coming to be misinformed. Beyond these examples involving false belief, the notion of being misinformed being dealt with here also encompasses cases where an agent is not aware of the false semantic content that they are holding. For example, an agent might hold some false semantic content that they have not rejected but do not believe since it is in an encrypted form which the agent cannot access.

As discussed above, the operator S corresponds to $\Box$ in the normal modal logic that results from adding $\Box\Box p \supset \Box p$ to the base normal modal logic **K**. This axiom corresponds to the density condition on the

[18]The possibility of capturing within a logical system the difference between merely holding true semantic content and being informed by holding a piece of actual information is open to further investigation.

accessibility relation: where w_1, w_2 and w_3 are worlds and R the accessibility relation, an accessibility relation is dense when the condition on frames is such that

$$w_1 R w_2 \Rightarrow \exists w_3 (w_1 R w_3 \wedge w_3 R w_2)$$

Therefore the logic for S is a normal modal logic characterised by dense frames.

3.7 Some Results

We begin by showing that the basic properties which we want valid, and which were valid in our earlier attempt to devise a logic of being misinformed, are also valid in this new system. Once again select proofs will be given using the standard tableaux method as found in [16]. The $\Box\Box p \supset \Box p$ axiom and density condition correspond to the following tree rule:

$$\begin{array}{c} w_1 R w_2 \\ | \\ w_1 R w_3 \\ w_3 R w_2 \end{array}$$

where w_3 is new to the tree branch

$\vdash \mathrm{M}p \supset \neg p$

$$\begin{array}{c} \neg(\mathrm{M}p \supset \neg p), w_1 \\ | \\ \mathrm{M}p, w_1 \\ \neg\neg p, w_1 \\ | \\ \mathrm{S}p \wedge \neg p, w_1 \\ | \\ \times \end{array}$$

$\vdash \mathrm{M}p \supset \neg\mathrm{M}\neg p$

$$\begin{array}{c} \neg(\mathrm{M}p \supset \neg\mathrm{M}\neg p), w_1 \\ | \\ \mathrm{M}p, w_1 \\ \neg\neg\mathrm{M}\neg p, w_1 \\ | \\ \mathrm{S}p \wedge \neg p, w_1 \\ \mathrm{S}\neg p \wedge \neg\neg p, w_1 \\ | \\ \times \end{array}$$

$\vdash \neg\mathrm{M}\top$

$$\begin{array}{c} \neg\neg\mathrm{M}\top, w_1 \\ | \\ \neg\neg(\mathrm{S}\top \wedge \neg\top), w_1 \\ | \\ \mathrm{S}\top \wedge \neg\top, w_1 \\ | \\ \mathrm{S}\top, w_1 \\ \neg\top, w_1 \\ | \\ \times \end{array}$$

$\vdash \mathrm{M}(\neg p \wedge q) \supset (\mathrm{M}p \supset \mathrm{M}q)$

$$\begin{array}{c}
\neg(\mathrm{M}(\neg p \wedge q) \supset (\mathrm{M}p \supset \mathrm{M}q)),\ w_1 \\
| \\
\mathrm{M}(\neg p \wedge q), w_1 \\
\neg(\mathrm{M}p \supset \mathrm{M}q), w_1 \\
| \\
\mathrm{M}p, w_1 \\
\neg\mathrm{M}q, w_1 \\
| \\
\mathrm{S}p \wedge \neg p, w_1 \\
\neg(\mathrm{S}q \wedge \neg q), w_1 \\
\mathrm{S}(\neg p \wedge q) \wedge \neg(\neg p \wedge q), w_1 \\
| \\
\mathrm{S}p, w_1 \\
\neg p, w_1 \\
\mathrm{S}(\neg p \wedge q), w_1 \\
\neg(\neg p \wedge q), w_1
\end{array}$$

$$\begin{array}{ccc}
\neg\neg p, w_1 & \multicolumn{2}{c}{\neg q, w_1} \\
| & & \\
\times & \neg\mathrm{S}q, w_1 & \neg\neg q, w_1 \\
 & \langle\mathrm{S}\rangle\neg q & | \\
 & | & \times \\
 & w_1 R w_2 & \\
 & \neg q, w_2 & \\
 & | & \\
 & p, w_2 & \\
 & \neg p, w_2 & \\
 & q, w_2 & \\
 & | & \\
 & \times &
\end{array}$$

The validity of the following formula requires the density condition on R:

$\vdash (\mathrm{SS}p \wedge \neg p) \supset \mathrm{M}p$

$$(\mathrm{SS}p \wedge \neg p), w_1$$

$$\neg \mathrm{M}p, w_1$$

$$\neg(\mathrm{S}p \wedge \neg p), w_1$$

$$|$$

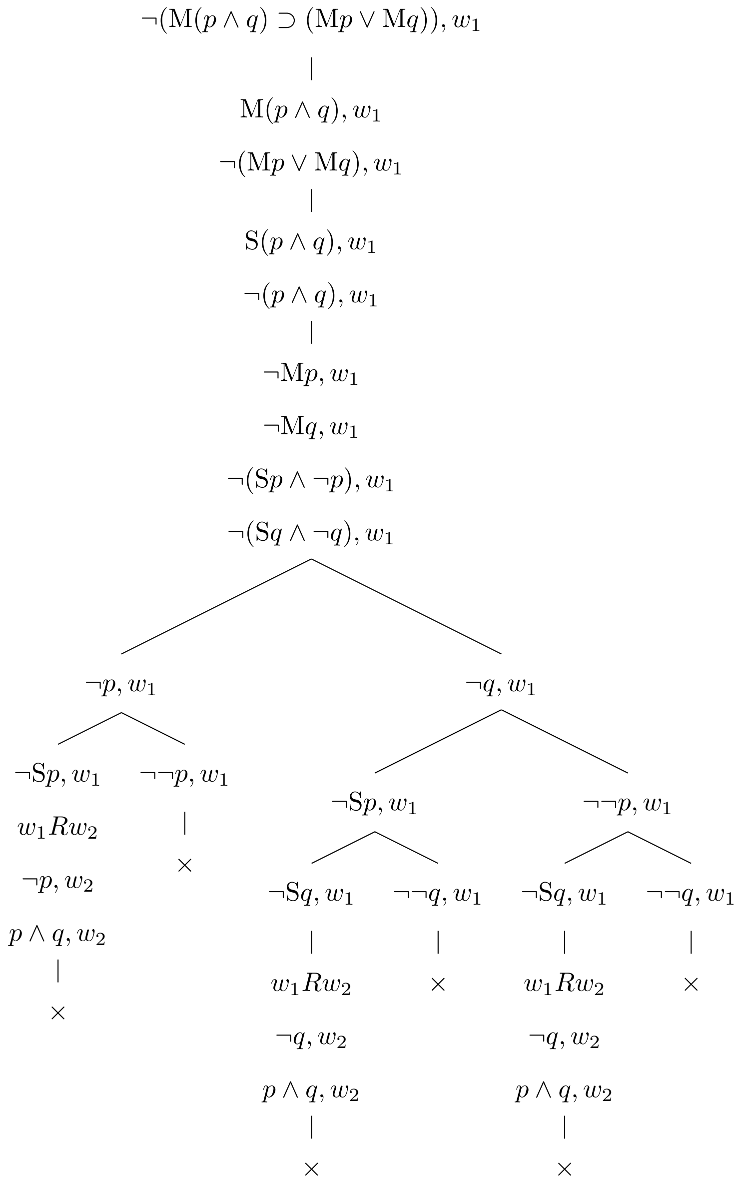
$$\mathrm{SS}p, w_1$$

$$\neg p, w_1$$

$$\begin{array}{cc}
\neg \mathrm{S}p, w_1 & \neg\neg p, w_1 \\
| & | \\
\langle \mathrm{S} \rangle \neg p, w_1 & \times \\
w_1 R w_2 & \\
\neg p, w_2 & \\
| & \\
w_1 R w_3 & \\
\mathrm{S}p, w_3 & \\
w_3 R w_2 & \\
p, w_2 & \\
| & \\
\times &
\end{array}$$

We also see that the two problematic results for the initial attempt discussed earlier are not problematic in this system. Firstly, the formula for Result 11 comes out valid.

$\vdash \mathrm{M}(p \wedge q) \supset (\mathrm{M}p \vee \mathrm{M}q)$



Secondly, the formula associated with Result 5, $\mathrm{M}(p \vee q) \supset \mathrm{M}p$, which was problematically valid for the previous system comes out invalid in the new system:

$\nvdash \mathrm{M}(p \vee q) \supset \mathrm{M}p$

$$\neg(\mathrm{M}(p \vee q) \supset \mathrm{M}p), w_1$$

$$|$$

$$\mathrm{M}(p \vee q), w_1$$

$$\neg\mathrm{M}p$$

$$|$$

$$(\mathrm{S}(p \vee q) \wedge \neg(p \vee q), w_1$$

$$\neg(\mathrm{S}p \wedge \neg p)$$

$$|$$

$$\mathrm{S}(p \vee q), w_1$$

$$\neg(p \vee q), w_1$$

$$\neg p, w_1$$

$$\neg q, w_1$$

$$\begin{array}{cc}
\neg\mathrm{S}p, w_1 & \neg\neg p, w_1 \\
\langle\mathrm{S}\rangle\neg p, w_1 & | \\
w_1 R w_2 & \times \\
\neg p, w_2 & \\
| & \\
p \vee q, w_2 & \\
\begin{array}{cc} p, w_2 & q, w_2 \\ | & \\ \times & \end{array} &
\end{array}$$

A counter-model read off the open branch in this tree is as follows:

- $W = \{w_1, w_2\}$
- $w_1 R w_2$
- $v_{w_1}(p) = 0, v_{w_1}(q) = 0, v_{w_2}(p) = 0, v_{w_2}(q) = 1$

Relative to w_1 it can be seen that whilst the agent has the semantic content that $p \vee q$ and $p \vee q$ is false, the agent does not hold the semantic content that p, thus does not hold the misinformation that p.

Here is a summary of other results, which can be compared to the previous table. Fortunately, every wanted formula comes out valid and every unwanted formula comes out invalid.

	Formula	Wanted?	Valid?		Formula	Wanted?	Valid?
1	$(\mathrm{M}p \vee \mathrm{M}q) \supset \mathrm{M}(p \vee q)$	no	$\times$	7	$(\mathrm{M}p \wedge \mathrm{M}q) \supset \mathrm{M}(p \wedge q)$	yes	$\surd$
2	$(\mathrm{M}p \vee \mathrm{M}q) \supset \mathrm{M}(p \wedge q)$	no	$\times$	8	$(\mathrm{M}p \wedge \mathrm{M}q) \supset \mathrm{M}(p \vee q)$	yes	$\surd$
3	$(\mathrm{M}p \vee \mathrm{M}q) \supset (\mathrm{M}p \wedge \mathrm{M}q)$	no	$\times$	9	$(\mathrm{M}p \wedge \mathrm{M}q) \supset (\mathrm{M}p \vee \mathrm{M}q)$	yes	$\surd$
4	$\mathrm{M}(p \vee q) \supset (\mathrm{M}p \vee \mathrm{M}q)$	no	$\times$	10	$\mathrm{M}(p \wedge q) \supset (\mathrm{M}p \wedge \mathrm{M}q)$	no	$\times$
5	$\mathrm{M}(p \vee q) \supset (\mathrm{M}p \wedge \mathrm{M}q)$	no	$\times$	11	$\mathrm{M}(p \wedge q) \supset (\mathrm{M}p \vee \mathrm{M}q)$	yes	$\surd$
6	$\mathrm{M}(p \vee q) \supset \mathrm{M}(p \wedge q)$	no	$\times$	12	$\mathrm{M}(p \wedge q) \supset \mathrm{M}(p \vee q)$	no	$\times$

Table 2: Summary of results

4 Combining Being Misinformed with Being Informed

Given these logics of being informed and misinformed, one obvious thing to do is combine them into a bimodal logic. Such a logic has two separate accessibility relations, R_I for the I operator and R_S for the S operator. The I operator behaves as it does in the logic of being informed. The S operator behaves as it does in the logic of holding semantic content. The one rule connecting I and S, $\mathrm{I}p \supset \mathrm{S}p$, corresponds to the following condition on accessibility: if $w_1 R_S w_2$ then $w_1 R_I w_2$. Here are some basic results in this system, all of them desirable:

1. $\vdash \mathrm{I}p \supset \neg\mathrm{M}p$
2. $\vdash \mathrm{M}p \supset \neg\mathrm{I}p$
3. $\nvdash \mathrm{I}\neg p \supset \mathrm{M}p$
4. $\nvdash \mathrm{M}p \supset \mathrm{I}\neg p$
5. $\vdash (\mathrm{I}\neg p \wedge \mathrm{S}p) \supset \mathrm{M}p$
6. $\nvdash \neg(\mathrm{I}p \wedge \mathrm{M}\neg p)$

Beyond these basic validations and invalidations, we can investigate formulas involving a richer interaction between I and M. Here is a case in point:

$\vdash (\mathrm{I}p \wedge \mathrm{M}(p \wedge q)) \supset \mathrm{M}q$

$$\neg((\mathrm{I}p \wedge \mathrm{M}(p \wedge q)) \supset \mathrm{M}q), w_1$$

$$|$$

$$\mathrm{I}p \wedge \mathrm{M}(p \wedge q), w_1$$

$$\neg \mathrm{M}q, w_1$$

$$|$$

$$w_1 R_I w_1$$

$$p, w_1$$

$$\mathrm{S}(p \wedge q) \wedge \neg(p \wedge q), w_1$$

$$\neg(\mathrm{S}q \wedge \neg q)$$

$$\begin{array}{ccc} \neg p, w_1 & & \neg q, w_1 \\ | & & \\ \times & & \end{array}$$

$$\begin{array}{cc} \neg \mathrm{S}q, w_1 & \neg\neg q, w_1 \\ | & | \\ \langle \mathrm{S} \rangle \neg q, w_1 & \times \\ w_1 R_S w_2 & \\ w_1 R_I w_2 & \\ \neg q, w_2 & \\ | & \\ \mathrm{S}(p \wedge q), w_1 & \\ p \wedge q, w_2 & \\ q, w_2 & \\ | & \\ \times & \end{array}$$

This result makes sense. Firstly, the agent holds the information that p, therefore they hold the semantic content that p and p is true. Secondly they hold the misinformation that $p \wedge q$, so they hold the semantic content that $p \wedge q$ and $\neg p \vee \neg q$. It follows from the agent also holding the semantic content that $p \wedge q$ that they also hold the semantic content that q and since p is true, q must be false. Therefore the agent holds the false semantic content (i.e. they are misinformed) that q.

Next is a formula, whose validity requires $\mathrm{I}p \supset \mathrm{S}p$.

$\vdash (\mathrm{I}p \wedge \mathrm{M}(\neg p \vee \neg q)) \supset \mathrm{M}\neg q$

$$\neg((\mathrm{I}p \wedge \mathrm{M}(\neg p \vee \neg q)) \supset \mathrm{M}\neg q), w_1$$

$$|$$

$$\mathrm{I}p \wedge \mathrm{M}(\neg p \vee \neg q), w_1$$

$$\neg\mathrm{M}\neg q, w_1$$

$$\mathrm{I}p, w_1$$

$$\mathrm{M}(\neg p \vee \neg q), w_1$$

$$|$$

$$w_1 R_I w_1$$

$$p, w_1$$

$$|$$

$$\neg(\mathrm{S}\neg q \wedge \neg\neg q), w_1$$

$$\mathrm{S}(\neg p \vee \neg q) \wedge \neg(\neg p \vee \neg q), w_1$$

$$\mathrm{S}(\neg p \vee \neg q), w_1$$

$$\neg(\neg p \vee \neg q), w_1$$

$$p, w_1$$

$$q, w_1$$

$$\begin{array}{cc}
\neg\mathrm{S}\neg q, w_1 & \neg\neg\neg q, w_1 \\
\langle \mathrm{S} \rangle q, w_1 & | \\
w_1 R_S w_2 & \times \\
q, w_2 & \\
\neg p \vee \neg q, w_2 & \\
\begin{array}{cc}
\neg p, w_2 & \neg q, w_2 \\
w_1 R_I w_2 & | \\
p, w_2 & \times \\
| & \\
\times &
\end{array} &
\end{array}$$

This result also makes sense. Imagine someone being informed that Rome is the capital of Italy. They are also misinformed that Rome is not the capital of Italy or Berlin is not the capital of Germany. Since Rome is the capital of Italy and they are informed of this, an inference they would make with the semantic content they hold is that Berlin is not the capital of Germany, a piece of misinformation.

Finally, if an agent is informed that they are misinformed that p, then they are informed that $\neg p$.

$\vdash \mathrm{IM}p \supset \mathrm{I}\neg p$

$$
\begin{array}{c}
\neg(\mathrm{IM}p \supset \mathrm{I}\neg p), w_1 \\
| \\
\mathrm{IM}p, w_1 \\
\neg\mathrm{I}\neg p, w_1 \\
\langle \mathrm{I} \rangle \neg\neg p, w_1 \\
| \\
w_1 R_I w_2 \\
w_1 R_I w_1 \\
p, w_2 \\
\mathrm{M}p, w_1 \\
\mathrm{M}p, w_2 \\
\mathrm{S}p \wedge \neg p, w_1 \\
\mathrm{S}p \wedge \neg p, w_2 \\
| \\
\neg p, w_1 \\
\neg p, w_2 \\
| \\
\times
\end{array}
$$

4.1 Being Told

Another potential extension is the introduction of a new operator, T_a, where, for want of a better term, $\mathrm{T}_a p$ stands for 'a is told that p'. T is defined as follows:

$$\mathrm{T}_a p =_{df} \mathrm{I}_a p \vee \mathrm{M}_a p$$

So firstly we have the following equivalences:

- $\vdash \mathrm{I}p \equiv \mathrm{T}p \wedge p$
- $\vdash \mathrm{M}p \equiv \mathrm{T}p \wedge \neg p$

In general, we can use results concerning I and M to help determine the validity/invalidity of a statement. For example, take the statement A, defined as $\mathrm{T}(p \wedge q) \supset \mathrm{T}(p \vee q)$, which reduces to $\mathrm{I}(p \wedge q) \vee \mathrm{M}(p \wedge q) \supset \mathrm{I}(p \vee q) \vee \mathrm{M}(p \vee q)$.

We know that $\vdash \mathrm{I}(p \wedge q) \supset \mathrm{I}(p \vee q)$ and $\nvdash \mathrm{M}(p \wedge q) \supset \mathrm{M}(p \vee q)$. So the validity/invalidity of A reduces to the validity/invalidity of the simpler $\mathrm{M}(p \wedge q) \supset \mathrm{I}(p \vee q) \vee \mathrm{M}(p \vee q)$, which is itself not valid. A simple counter-model is as follows:

- $W = \{w_1, w_2\}$
- $w_1 R_I w_1, w_2 R_I w_2, w_1 R_I w_2, w_2 R_I w_1$
- $v_{w_1}(p) = 1, v_{w_1}(q) = 0, v_{w_2}(p) = 0, v_{w_2}(q) = 0$

Here are some other results:

- $\vdash (\mathrm{T}p \wedge \mathrm{T}q) \supset \mathrm{T}(p \vee q)$
- $\vdash \mathrm{T}(p \wedge q) \supset (\mathrm{T}p \vee \mathrm{T}q)$
- $\vdash (\mathrm{T}p \wedge \mathrm{T}q) \supset \mathrm{T}(p \wedge q)$

The last of these results requires the axiom $\mathrm{I}p \supset \mathrm{S}p$. If it were not present, the following would serve as a counter-model:

- $W = \{w_1, w_2, w_3\}$
- $w_1 R_I w_1, w_2 R_I w_2, w_3 R_I w_3, w_1 R_I w_2, w_1 R_S w_3$
- $v_{w_1}(p) = 0, v_{w_1}(q) = 1, v_{w_2}(p) = 0, v_{w_2}(q) = 1, v_{w_3}(p) = 1, v_{w_3}(q) = 0$

With this counter-model, $\mathrm{I}q$ and $\mathrm{M}p$ but neither $\mathrm{I}(p \wedge q)$ nor $\mathrm{M}(p \wedge q)$. If the constraint for $\mathrm{I}p \supset \mathrm{S}p$ is added, then $\neg\mathrm{S}p$ so $\neg\mathrm{M}p$.

T is not a normal operator: $\nvdash \mathrm{T}(p \supset q) \wedge \mathrm{T}p) \supset \mathrm{T}q$. A simple counter-model is as follows:

- $W = \{w_1, w_2\}$
- $w_1 R_I w_1, w_2 R_I w_2, w_1 R_I w_2, w_2 R_I w_1$
- $v_{w_1}(p) = 0, v_{w_1}(q) = 1, v_{w_2}(p) = 0, v_{w_2}(q) = 0$

How can we explain the possibility of having $\mathrm{T}(p \supset q)$ and $\mathrm{T}p$ but not $\mathrm{T}q$? Well, to begin with, we can reduce things here to terms involving I and M. At w_1, $\mathrm{I}(p \supset q)$ and $\mathrm{M}p$ but $\neg(\mathrm{I}q \vee \mathrm{M}q)$. Is it plausible for an agent to be informed that $p \supset q$, misinformed that p and neither informed nor misinformed that q?

Firstly, a breakdown of what is happening here. Since $\mathrm{I}(p \supset q)$, it is the case that $\mathrm{I}(\neg p \vee q)$ and $\neg p \vee q$. Since $\mathrm{M}p$, it is the case that $\mathrm{S}p$ and $\neg p$. From here, q can be true or it can be false and in this case it is true, so it cannot be misinformation and the agent cannot be misinformed that q. But neither does it follow that the agent is informed that q. They do have the true semantic content that q, since they have the semantic content that $p \supset q$ and the semantic content that p. But as discussed, holding true semantic content is not sufficient for being informed and the agent's informational accessibility relation accesses a state in which q is not information. Also, in this case the agent oddly but not implausibly has the information that $\neg p$ and the misinformation that p.

Now, can all this be associated with an actual scenario? Although not straightforward, the following type of example seems to fit. We have an agent, say John, who is informed by an information carrying source that $\neg p$. From this it logically follows that he is also informed that $\neg p \vee q$. He is also unknowingly misinformed that p, so $\neg p$ is the case. Now, although it so happens that q is true and John holds the semantic content that q via holding the semantic contents that $p \supset q$ and p, the holding of this true semantic content would not be via an information source; its deduction would involve a false premiss, namely p. At any rate, beyond this base explanation, if the agent in question was aware that they held the information that $\neg p$ then they would no longer be misinformed that p, they would no longer hold this misinformation.

5 Conclusion

The purpose of this paper has been to investigate the notion of misinformation and develop a modal logic of being misinformed. Such a task falls under the general enterprise of investigating the notion of misinformation in tandem with the notion of information. Whilst a logic of being misinformed naturally complements a logic of being informed, this paper has shown that in some ways misinformation is perhaps a little trickier than information and that an attempt to logically capture the relation of being misinformed has some uniquely distinctive concerns that sets it apart from attempts to logically capture the relation of being informed. One side aspect of this paper worth pointing out is that not only did the development of a logic of being misinformed require a sufficient account of misinformation to begin with, but that considerations in the development of the logic aided in clarification and helped shape the final understanding of misinformation. After one initial problematic attempt, an approach was settled upon whereby 'being misinformed' was defined in terms of holding false semantic content. Thus an initial sub-task of this approach was the selection of a suitable modal logic to capture the relation of 'holding semantic content'. It was stipulated that the possession of or holding of semantic content by an agent at least implies that the agent has not rejected the truth or accepted the falsity of the semantic content. It was further determined here that the holding of semantic content by an agent implies that the agent presumes its truth when they are aware that they are holding the content. Finally, the obtaining of a logic of being misinformed leads to combining it with a logic of being informed in which interaction between the two operators can be investigated and a 'told' operator can be defined.

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