The Logic Of Being Informed

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It is well established that the states of knowledge and belief have been captured using systems of modal logic. Referred to respectively as epistemic and doxastic modal logics, they have been studied extensively in the literature. In a relatively recent paper entitled 'The Logic Of Being Informed' [6], Luciano Floridi does the same for the state of being informed, giving a logic of being informed also based on modal logic. In this *information logic* (**IL**), the modal operator I_a stands for 'agent a is informed that' so that the statement $\mathrm{I}_a p$ stands for 'a is informed that p' or 'a holds the information that p'.[1]

1 The Logic Of Being Informed

Information is a pluralistic concept, and can be understood in a variety of ways. In developing a logic of being informed, the sort of information being dealt with is information as semantic content, semantic content that is held by an informational agent or *informee*. If σ is an instance of information, understood as semantic content, then:

(GDI.1) σ consists of one or more data;

(GDI.2) the data in σ are well-formed;

(GDI.3) the well-formed data in σ are meaningful.

[1] Of course, whether or not epistemic and doxastic logics adequately capture knowledge and belief is open to debate. But as Floridi points out, the task he sets himself is to determine an information logic, different from epistemic logic and doxastic logic, that formalises the relation "a is informed that p" equally well. The keyword here is "equally" not "well". He argues that **IL** can do for "being informed" what epistemic logic does for "knowing" and what doxastic logic does for "believing". If one objects to the last two, one may object to the first as well, yet one should not object to it more.

So data are the stuff of which information is made; information cannot be dataless, but, in the simplest case, it can consist of a single datum. A general definition of a datum is:

A datum is a putative fact regarding some difference or lack of uniformity within some context.[2]

Leaving aside further discussion on the nature of data, some examples will help to clarify the gist of this definition. Take a single sheet of unmarked white paper. It is an example of complete uniformity; each unit of the paper's surface is the same as every other unit.[3] As it is, there is no datum associated with this sheet. If a black marker were used to place a black dot in the middle of the sheet, then there would be a lack of uniformity. The white background plus the black dot would constitute the datum.[4]

Or as another example, consider a unary alphabet, consisting of the symbol 0. Any source that continuously emits symbols from this alphabet is not emitting data, for there is no lack of uniformity in its output. However, if the alphabet were expanded to include the symbol 1 as well as the symbol 0, then it would be possible for the source to emit data, by using both instances of the 0 symbol and instances of the 1 symbol.

In (GDI.2), 'well-formed' means that the data are composed according to the rules (syntax) governing the chosen system, code or language being analysed. Syntax here is to be understood generally, not just linguistically, as what determines the form, construction, composition or structuring of something. The string 'the an two green four cat !?down downx' is not well-formed in accordance with the rules of the English language, so therefore cannot be an instance of semantic information in the English language. Or, to take another example, the string '$A \neg B$' is not well-formed in accordance with the rules of the language of propositional logic, so therefore cannot be an instance of semantic information in propositional logic.

[2]See http://plato.stanford.edu/entries/information-semantic/#1.3 for discussion of this definition

[3]Whatever a unit might be measured in, pixels, millimetres, etc. Also, when comparing units, the attribution of sameness is based only on a certain property, namely that each unit is in its original state of unmarked whiteness. In certain ways each unit might differ. For example, each unit is in a different part of the sheet of paper, some units might be smoother than others. In this case, we are talking about the state of each unit in terms of its marking.

[4]This involves the notion of Taxonomic Neutrality. A datum is a relational entity. Neither of these two *relata*, the black dot or the white background, is the datum. Rather both, along with the fundamental relation of inequality between the dot and the background constitute the datum.

In (GDI.3), ‘meaningful’ means that the well-formed data must comply with the meanings (semantics) of the chosen system, code or language in question. For example, the well-formed string ‘Colourless green ideas sleep furiously’ cannot be semantic information in the English language because we may say (without getting into a debate about theories of meaning) that it is meaningless; it does not correspond to anything. Finally, an example of a string which fulfills (GDI.1), (GDI.2) and (GDI.3) is ‘The native grass grew nicely in spring’.

There are two main types of information, understood as semantic content: factual and instructional. The type which we are interested in is factual semantic information.[5] For semantic information to be factual, it needs to be about some state of affairs, about some fact. Factual information comes in a variety of forms; here are some examples:

- A map of Europe contains the factual information that Germany is north of Italy, in the language of cartography. The data that this information is made of is identified with the sheet of paper on which the map is printed plus the various markings on the page. This data is well-formed; among other things, the North-South-East-West coordinates are correctly positioned and no countries are marked as overlapping each other. Finally, this data is meaningful. Each part of the paper, contained in a thick black line and shaded in a certain colour corresponds or refers to a country. Thin blue lines mean rivers, etc.

- A person’s nod contains the factual information that they are in agreement, in certain human body languages. The data that this information is made of is indentified with the variation in head position. This data is well-formed; head movement is a legitimate expression in the language. This data is also meaningful; this particular expression means ‘yes’ or ‘positive’.

- The content of an Encyclopaedia Britannica entry on Italy will contain the information that Rome is the capital of Italy, in the language of English. The data that this information is made of is indentified with the varied string of English alphabet symbols that constitute the entry. This data is well-formed as it accords with the syntax of the English language. It is also meaningful to an English language reader.

[5] For more on instructional information, see http://plato.stanford.edu/entries/information-semantic/#3.1

Ultimately, these various forms of semantic information are reducible to propositional form, or propositional expression. If p is factual information, then it can be expressed in the form 'the information that p'. This leads to an identification of information with propositions.[6]

So Floridi is discussing information in the intuitive sense of declarative, objective and semantic content that p or about f. Furthermore, the focus is on the statal condition into which an agent a enters, once a has acquired the information that p. When John consults a map of Europe to learn about the respective positions of Germany and Italy, he acquires the information that Germany is north of Italy. When Harry nods to John in response to John's question 'are you hungry Harry', John acquires the information that Harry is hungry. When John consults the Encyclopaedia Britannica entry on Italy to find out what its capital city is, he acquires the information that Rome is the capital of Italy.

So what is required is a logic of being informed (i.e. holding the information), in contrast to a logic of being informative or a logic of becoming informed.

The system of logic suggested by Floridi to formally capture the relation of 'being informed' is the normal modal logic system commonly referred to as **KTB** (also known as **B**, **Br** or **Brouwer's** system). So **IL** is constructed as an informational reading of **KTB**.

The modal operator corresponding to $\Box$ is interpreted as 'is informed that'. Replacing the symbol $\Box$ with the symbol I, and including a subscript reference to the agent involved, we get:

$$\mathrm{I}_a p\text{: } a \text{ is informed that } p$$

Floridi defines the accompanying modal operator corresponding to $\Diamond$ in the standard following way

$$\mathrm{U}_a p =_{def} \neg \mathrm{I}_a \neg p$$

which can be read as a is uninformed (is not informed, does not hold the information) that $\neg p$; or for all a's information (given a's information base), it is possible that p.

Furthermore:

[6] For more discussion of this identification, see my paper 'Information: Its Quantification and Alethic Nature'

a's information base can be modelled by representing it as a dynamic set D_a of sentences of a language L. The intended interpretation is that D_a consists of all the sentences, i.e. all the information, that a holds at time t. We then have that $I_a p$ means that $p \in D_a$, and $U_a p$ means that p can be uploaded in D_a while maintaining the consistency of D_a, that is, $U_a p$ means $\Diamond(p \in D_a)$.

It is important to note that Floridi's approach here is syntactical, rather than semantic in nature. Axioms are assessed for suitability based how they accord with considerations of the notion of being informed. A semantic approach on the other hand would provide an interpretation of the Kripke-style semantics involved. For example, with the standard epistemic interpretation of Kripke-style semantics, a knows that p means that in all possible worlds compatible with what a knows, it is the case that p. No such account of informational accessibility is provided by Floridi.[7]

In arriving at his selection of the system **KTB**, Floridi considers 11 modal logic axiom schemata and systematically justifies his inclusion of some and exclusion of others. Here is a list of them, followed by some commentary:

[7]Patrick Allo revisits and revises the logic of being informed in [1], where he discusses among other things an informational interpretation of the accessibility relation.

Label	Definitions of Axiom Schemata	Name of the Axiom or Corresponding NML	Frame Property	Part of **IL**
A_1	$A \to (B \to A)$	1^{st} axiom of PC		$\surd$
A_2	$(A \to (B \to C)) \to ((A \to B) \to (A \to C))$	2^{nd} axiom of PC		$\surd$
A_3	$(\neg B \to \neg A) \to (A \to B)$	3^{rd} axiom of PC		$\surd$
A_4	$\Box A \to A$	KT or M, K2, veridicality	Reflexive	$\surd$
A_5	$\Box(A \to B) \to (\Box A \to \Box B)$	K, distribution, deductive cogency	Normal	$\surd$
A_6	$\Box A \to \Box\Box A$	4, S4, K3, KK, reflective thesis or positive introspection	Transitive	$\times$
A_7	$A \to \Box\Diamond A$	KTB, B, Br, Brouwer's axiom or Platonic thesis	Symmetric	$\surd$
A_8	$\Diamond A \to \Box\Diamond A$	S5, reflective, Socratic thesis or negative introspection	Euclidean	$\times$
A_9	$\Box A \to \Diamond A$	KD, D, consistency	Serial	$\surd$
A_{10}	$(\Box(A \to B) \to (\Box(B \to C) \to \Box(A \to C)))$	Single agent transmission		$\surd$
A_{11}	$\Box_x\Box_y A \to \Box_x A$	K4, multiagent transmission,or Hintikka's axiom		$\surd$

1.1 IL Satisfies A_1, A_2, A_3, A_5

IL is trivially assumed to satisfy the axioms A_1 - A_3. As for axiom A_5, which gives the normal modal logic **K**, this is straightforwardly added. Like many other cognitive relations, 'being informed' is distributive. If an agent a holds the information that $p \to q$, then, if a holds the information that p, a also holds the information that q.

I will briefly mention one result of the system **K** which could be construed as problematic in the context of capturing the relation of being informed. It concerns the closure of information, represented by the valid inference:

$$\mathrm{I}(p \supset q), \mathrm{I}p \vdash \mathrm{I}q$$

Fred Dretske has famously argued against epistemic closure. In short, since the modes of gaining, preserving or extending knowledge, such as perception, testimony, proof, memory, indication, and *information* are not individually closed, neither is knowledge [7].

Also, Floridi writes:

> If an agent a is informed that $p \rightarrow q$, then, if a is informed that p, a is also informed that q. Note that although this is entirely uncontroversial, it is less trivial. Not all "cognitive" relations are distributive. "Knowing", "believing" and "being informed" are, as well as "remembering and recalling". ... However, "seeing" and other experiential relations, for example, at not: if an agent a sees (in a non metaphorical sense) or hears or experiences that $p \rightarrow q$, it may still be false that, if a sees (hears, etc.) p, a then also sees (hears, etc.) q.

Floridi's comments here provoke a question. If such non-closed experiential relations are modes of information acquisition, how is the acquired information closed?

1.2 Consistency and Truth: IL satisfies A_9 and A_4

Firstly the inclusion of the weaker A_9 is justified. It says that the information holding agent is consistent, in the sense that if an agent holds a piece of information p, then the agent cannot be informed of $\neg p$. Not because the agent cannot hold $\neg p$, but because $\neg p$ fails to qualify as information. An agent can store both the *semantic contents* or *data* p and $\neg p$, but only one of them will count as information.

This leads to the stronger thesis captured by A_4, which represents the veridicality thesis, that for something to count as information, for a proposition to be informative, it must be true. The axiom **T** represents the so-called veridicality condition, that A being information implies that A is true.[8]

[8]For a discussion of why data must be true to count as information, see my paper 'Information: Its Quantification and Alethic Nature'.

1.3 No reflectivity: IL does not satisfy A_6, A_8

The absence of the axioms A_6 and A_8 from this system of information logic sets it apart from standard epistemic logics. These axioms do not qualify for inclusion in a logic of being informed because informational agents need not be introspective. Whilst 'believing' and 'knowing' are arguably mental states that possess a reflective transparency, the relation of being informed does not require a mental or conscious state. An artificial informational agent can possess the information that p without possessing the information that they posses this information. Even humans can be informed of something without being aware they have this information. So $\mathrm{I}p \supset \mathrm{II}p$ does not hold. Similarly for $\mathrm{U}p \supset \mathrm{IU}p$, which is equivalent to $\neg\mathrm{I}\neg p \supset \mathrm{I}\neg\mathrm{I}\neg p$; if an agent does not have the information that not-p, they need not be informed of their uninformed state.

1.4 Transmissibility: IL satisfies A_{10} and A_{11}

A_{10} is a theorem of all Normal Modal Logics. In the context of the logic of being informed, it simply captures the transitivity of information holding. A_{11} also makes perfect sense. If agent a holds the information that agent b holds the information that p, then a also holds the information that p. A derivation of this axiom requires usage of only a few core agreeable principles: (1) the veridicality of information (2) that agents are informed of the veridical nature of information (3) the distributivity of being informed.

$$\vdash \mathrm{I}_b p \supset p \tag{1}$$

$$\vdash \mathrm{I}_a(\mathrm{I}_b p \supset p) \tag{2}$$

$$\vdash \mathrm{I}_a(\mathrm{I}_b p \supset p) \supset (\mathrm{I}_a\mathrm{I}_b p \supset \mathrm{I}_a p) \tag{3}$$

$$\vdash \mathrm{I}_a\mathrm{I}_b p \supset \mathrm{I}_a p \tag{4}$$

1.5 Constructing the information base: IL satisfies A_7

Although a justification for the inclusion of A_7 in the context of information logic is not obvious, some deliberation about this axiom will show that it makes sense. To start with, consider it in the form $p \supset \mathrm{I}_a\neg\mathrm{I}_a\neg p$. Floridi writes

> **IL** satisfies A_7 in the sense that, for any true p, the informational agent a not

only cannot be informed that $\neg p$ (because of A_4), but now is also informed that a does not hold the information that $\neg p$. [6, p. 16]

This axiom is saying something about the makeup of informational agents. Basically, for all data which an agent does not have stored in its information base, the agent is informed that they do not have this data. If p is true it is information, and $\neg p$ is false so it is not information. Since $\neg p$ is not information, then it cannot be a part of an agent's information base, a fact of which the agent is informed.

Given A_7, the formula $\mathrm{U}_a\mathrm{I}_a p \supset p$ is derivable. Apparently the provability of this formula is a potential objection to the inclusion of A_7. "Ontologically, this is known to be a rather controversial result. Yet informationally, $\mathrm{U}_a\mathrm{I}_a p \supset p$ has a very intuitive reading" [6, p. 17].

In my efforts to comprehend this statement in informational terms, I found that a good way to understand it begins by negating it, and seeing that satisfaction of the resulting statement would not make sense.

$\neg(\mathrm{U}_a\mathrm{I}_a p \supset p)$

$\neg(\neg\mathrm{I}_a\neg\mathrm{I}_a p \supset p)$

$\neg\mathrm{I}_a\neg\mathrm{I}_a p \wedge \neg p$

$\mathrm{U}_a\mathrm{I}_a p \wedge \neg p$

What would it mean for p to be false and $\mathrm{U}_a\mathrm{I}_a p$ to be true? (1) if p is false, then as discussed above an agent cannot hold the information that p. Since the possibility of an agent holding p as information is ruled out, then p cannot be in the agent's information base (2) Since p is false, then it should not be possible that for all a's information a holds the information that p. Since $\mathrm{U}_a\mathrm{I}_a p \wedge \neg p$ affirms (1) but does not affirm (2), it should not be satisfiable, therefore $\mathrm{U}_a\mathrm{I}_a p \supset p$ is valid.

$\mathrm{U}_a\mathrm{I}_a p \supset p$ can replace $p \supset \mathrm{I}_a\mathrm{U}_a p$ as the characterising axiom of **KTB**. Floridi actually adopts it instead in his list of axioms for the resulting system.

Thus the normal modal logic which results, consists of the following axiom schemata and rules:

- $p \supset (q \supset p)$
- $(p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$

- $(\neg p \supset \neg q) \supset (q \supset p)$
- $\mathrm{I}p \supset p$
- $\mathrm{I}(p \supset q) \supset (\mathrm{I}p \supset \mathrm{I}q)$
- $\mathrm{UI}p \supset p$
- $\vdash p, \vdash p \supset q \Rightarrow \vdash q$ (Modus Ponens)
- $\vdash p \Rightarrow \vdash \mathrm{I}p$ (Rule of Necessitation)

Regarding the rules of the system, we may trivially assume acceptance of Modus Ponens.

The Rule of Necessitation translates to the implication that an agent is informed about all theorems provable in the system. The epistemic logic equivalent of this phenomenon, what has been termed the 'logical omniscience' problem, has received its fair share of attention. Unless one is talking of an ideal epistemic agent, aware of all logical truths, it is problematic for an epistemic logical system which entails that an agent's knows all logical truths. Whilst it is thus very understandable to try and avert the logical omniscience issue, the information logic equivalent I would argue does not need to be averted with the same urgency, if at all, for it can be construed in an acceptable way.

In the cases of knowledge and belief, the fact that conscious mental states are required precludes the realistic possibility of logical omniscience. With information however, we are not talking about something which is present and accessible within a particular mental space of an agent. We are talking about the objective information that is contained within a system.

If an agent knows that p and knows that $p \supset q$, then they will only come to know that q once they have made that inference. On the other hand, if an agent has the information that p and the information that $p \supset q$, then they have the information that q, prior to and independent of the inference being made by them. So it is reasonable to say that in an informational agent equipped with the information of axioms and inference rules associated with a logical system, they are implicitly in possession of all the information that is entailed by the system.

A further supporting consideration is the fact that standard semantic accounts of information do not count tautologies as informative.

With Bar-Hillel and Carnap's account of semantic information [2], using some probability measure pr, two measures of information, $cont$ and inf are provided, such that:

$$cont(A) =_{df} 1 - pr(A)$$

and

$$inf(A) =_{df} -log_2(pr(A))$$

When the probability of A is 1, $cont(A) = 1 - 1 = 0$ and $inf(A) = -log_2(1) = 0$.

In another example, Fred Dretske's definition of informational content given in *Knowledge and the Flow of Information* [4] is

A signal r carries the information that s is F = The conditional probability of s's being F, given r (and k), is 1 (but given k alone, less than 1).

Here k stands for what the receiver already knows concerning the possibilities from the source. Once again, if the signal carries tautological content, then even without any signal and given k alone, the conditional probability would be 1. Since it would not be less than 1, the signal carries no information.

These accounts of information are based on the *Inverse Relationship Principle* [3], according to which the information carried by an event or structure is inversely proportional to its probability. Even an alternative approach to information such as Floridi's theory of strongly semantic information [5], which is based on truthlikeness measures, assigns an informativeness of 0 to tautologies.

The stipulation that tautological structures are not informative can be nicely accommodated by the idea that agents already possess such information by default, and since whatever information an agent already possesses can no longer be informative, tautologies are not informative.[9]

A weakening of the Rule of Necessitation could be in order though. The Weak Rule of Necessitation is as follows:

[9]There is a sense in which logical, mathematical and analytic truths can be informative, but this sense of informativeness differs to that being discussed here.

$$\text{If } p \text{ is a theorem of PC then I}p$$

So in replacing the standard Rule of Necessitation with its weak counterpart, it would still be the case that agents are informed of all tautologies. What would change is that being informed about tautologies would no longer be iterative, it would not follow that agents are informed of the fact that they are informed of a tautology.[10] An adoption of the Weak Rule of Necessitation would be based on the non-reflectivity of being informed, which was argued for in rejecting the positive introspection (A_6) and negative introspection (A_8) axioms.

[10] The weak rule of necessitation is associated with non-normal modal logics.

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