Comments on Niiniluoto's Min-Sum Measure

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1 Niiniluoto on Truthlikeness

Ilkka Niiniluoto is a preeminent figure in the truthlikeness (or verisimilitude) research program. In particular his *Truthlikeness* [3] is a dense and significant contribution.

His approach to truthlikeness falls under the *similarity approaches*. They involve defining some distance function Δ such that $\Delta(A, C_*) \in [0, 1]$, where A is any statement and C_* is the true state description. The result of this distance calculation is then used to calculate truthlikeness: $\mathrm{Tr}(A, C_*) = 1 - \Delta(A, C_*)$.

Niiniluoto's favoured measure is the 'min-sum' measure $\Delta^{\gamma\lambda}_{\mathrm{ms}}$, which is a combination of the 'min' and 'sum' measures. Following are the definitions for these measures; but firstly, establishment of some terms which will be used:

- $\Delta(w_i, w_j)$ calculates the distance between states w_i and w_j. This is the sum of atomic differences multiplied by the atomic weight ($\frac{1}{n}$), where n is the number of propositions in the space.
- w_* is the actual state, that corresponds to the true state description C_*.
- W_A is the set of states in which A is true.
- $\mathbf{B}$ is the set of all states in the logical space.

Here are the distance functions:

- $\Delta_{\mathrm{min}}(A, C_*)$ = the minimum of the distances $\Delta(w_a, w_*)$ with $w_a \in W_A$.
- $\Delta_{\mathrm{sum}}(A, C_*)$ = the sum of all distances $\Delta(w_a, w_*)$ with $w_a \in W_A$, divided by the sum of all distances $\Delta(w_b, w_*)$ with $w_b \in \mathbf{B}$.
- $\Delta^{\gamma\lambda}_{\mathrm{ms}}(A, C_*) = \gamma\Delta_{\mathrm{min}}(A, C_*) + \lambda\Delta_{\mathrm{sum}}(A, C_*)$ for some two weights γ and λ with $0 \leq \gamma \leq 1$ and $0 \leq \lambda \leq 1$

The following values for γ and λ will be used. They were experimented with and gave reasonable results. Also, as Niiniluoto writes, "in many applications, choosing [γ] to be equal to 2λ gives intuitively reasonable results" [4].

- $\gamma = 0.89$
- $\lambda = 0.44$.

Here is an example using that measure:

Example Take the canonical weather framework example:

State	h	r	w
w_1	T	T	T
w_2	T	T	F
w_3	T	F	T
w_4	T	F	F
w_5	F	T	T
w_6	F	T	F
w_7	F	F	T
w_8	F	F	F

- w_1 is the true state. So $C_* = h \wedge r \wedge w$
- Let A be the statement $h \wedge (r \vee \neg w)$. This holds in states w_1, w_2, w_4
- $\Delta_{\text{min}}(A, C_*) = \Delta(w_1, w_1) = 0 \times \frac{1}{3} = 0$
- $\Delta_{\text{min}}(A, C_*) = \frac{3}{12} = \frac{1}{4}$
- $\Delta^{\gamma\lambda}_{\text{ms}}(A, T) = 0\gamma + \frac{\lambda}{4}$
- $\text{Tr}(A) = 1 - \Delta^{\gamma\lambda}_{\text{ms}}(A, C_*) = 1 - \frac{\lambda}{4} = 1 - \frac{0.44}{4} = 0.89$

2 Truthlikeness and Belief Revision

Within the last few years there has been interest in investigating the relationship between the research programs of truthlikeness and belief revision. Niiniluoto himself was possibly the first to start looking into this [2, 5]. For the truthlikeness part Niiniluoto used his preferred min-sum measure and for the belief revision part he employed the standard AGM framework.

In a recent paper [1] the relationship between (AGM) belief change and a particular approach to verisimilitude (truthlikeness), namely the 'basic feature approach' (BF-approach), is investigated. A few theorems are given, which it is suggested "hold for any plausible verisimilitude measure defined on propositional languages" [1, p. 58.].

However some but not all of these theorems hold for Niiniluoto's min-sum measure. In the following examples AGM sphere semantics were used for revision procedures (as outlined in [2]), with the standard Δ function being used to order states. $T + A$ denotes the expansion of theory T by input A and $T * A$ the revision.

In the following theorems, both A and T are c-propositions [1, p. 50.], which are basically statements in conjunctive normal form, where each conjunct is of the form p or $\neg p$, with p being an atom. These conjuncts are called literals and denoted with $\pm p$.

Here is Theorem 10. of [1, p. 57.]:

Theorem 2.1. Suppose that A is true. Then:

1. $\text{Tr}(T + A) > \text{Tr}(T)$
2. $\text{Tr}(T * A) > \text{Tr}(T)$

This theorem holds for Niiniluoto's min-sum measure:

Proof. The expansion of T with a true c-proposition $\pm p_1 \wedge \pm p_2 \wedge ... \pm p_k$ can be seen as the successive expansion of T with each conjunct of A, all of which are true.

1. The expansion of T by a true $\pm p$ is going to reduce the number of states corresponding to T, with each successive addition of a $\pm p$ halving the set of corresponding states. The eliminated states are each going to have a higher valued corresponding state that remains in the collection of states. For each eliminated state, the state corresponding to it will simply be the state that is the same on every atom except for the atom of the basic claim being added. In the eliminated state it will be false and in the remaining state it will be true.

   It follows that when a set of states is so halved, the state closest to the true state will always remain. So the min measure will never increase. The sum measure will also never increase, since the number of total differences can only decrease. Therefore min+sum will never increase, so Tr(A) = 1 - (min+sum) will never decrease.

2. For revision, if a basic proposition $\pm p$ does not conflict with T then it is simply a case of expansion. If p does conflict with T, then all instances of p in the collection of states are replaced by the true p. This means that both the min measure and the sum measure will decrease.

□

For all the following examples, the true state is that in which each atom p_i is true.

Here is Theorem 11. of [1, p. 57.]:

Theorem 2.2. Suppose that A_{cT} and A_{xT} are verisimilar. Then:

1. $\text{Tr}(T + A) > \text{Tr}(T)$
2. $\text{Tr}(T * A) > \text{Tr}(T)$

Both of these inequalities for the most part hold, though there are some exceptions:

Proof. If A_{cT} and A_{xT} are verisimilar then more than half of their b-claims are true.

1. Take a 4-proposition logical space consisting of propositions $\{p_1, p_2, p_3, p_4\}$.

- $T = p_1$
- $A = p_2 \wedge p_3 \wedge \neg p_4$
- $T + A = p_1 \wedge p_2 \wedge p_3 \wedge \neg p_4$

$\text{Tr}(p_1 \wedge p_2 \wedge p_3 \wedge \neg p_4) = 0.764 < \text{Tr}(p_1) = 0.835$

2. Take an 8-proposition logical space, consisting of propositions $\{p_1 - p_8\}$.

- $T = \neg p_1 \wedge \neg p_2 \wedge p_3$
- $A = p_1 \wedge p_2 \wedge \neg p_3 \wedge p_4 \wedge p_5 \wedge p_6 \wedge \neg p_7 \wedge \neg p_8$
- A_{cT} is $p_1 \wedge p_2 \wedge \neg p_3$
- A_{xT} is $p_4 \wedge p_5 \wedge p_6 \wedge \neg p_7 \wedge \neg p_8$
- $T * A = p_1 \wedge p_2 \wedge \neg p_3 \wedge p_4 \wedge p_5 \wedge p_6 \wedge \neg p_7 \wedge \neg p_8$

$\text{Tr}(T * A) = 0.665 < \text{Tr}(T) = 0.716$

□

Here is Theorem 12. of [1, p. 58.]:

Theorem 2.3. Suppose that A is completely false: Then:

1. $\text{Tr}(T + A) < \text{Tr}(T)$
2. $\text{Tr}(T * A) < \text{Tr}(T)$

Both of these inequalities for the most part hold, though there are some exceptions:

Proof. 1. Take a 12-proposition logical space, consisting of propositions $\{p_1 - p_{12}\}$. Let:

- $T = p_1$
- $A = \neg p_2$
- $T + A = p_1 \wedge \neg p_2$

$\text{Tr}(T) = 0.798 < \text{Tr}(T + A) = 0.816$

2. Take a 24-proposition space with propositions, $\{p_1 - p_{24}\}$. Let:

- $T = p_1$
- $A = \neg p_1 \wedge \neg p_2$
- $T * A = \neg p_1 \wedge \neg p_2$

$\text{Tr}(T) = 0.789 < \text{Tr}(T * A) = 0.807$

□

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