Knowing that p and knowing that p is true [Draft]

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Abstract

An issue that is raised against externalist accounts of knowledge such as those of Dretske and Nozick is that they entail certain 'abominable conjunctions'. In a recent paper [17], Michael Veber attempts to buttress the argument from abomination by adding a new type of abomination to the list: logical abominations. In this paper I argue that whilst such accounts do indeed entail abominable conjunctions their externalist nature means that they do not actually entail these logical abominations. I also take one of the abominations Veber uses as an opportunity to defend Dretske's account against claims that it violates the principles of Addition Closure and Conjunction Distribution.

Keywords: Knowledge, Externalism, Epistemic Closure, Dretske, Abominable Conjunction

1 An Outline of Dretske's Account of Knowledge

Fred Dretske, one of the pioneers of externalism and relevant alternative accounts in epistemology, developed a highly influential account of knowledge starting off with the idea that knowledge is belief grounded in conclusion reasons [8]. He would later give a complete embodiment of these ideas via his informational account of knowledge [9]. In short, according to Dretske knowledge is defined as information-caused belief. Information carrying and belief causing signals provide the externalist component and a veridical conception of information ensures that such beliefs are true. For Dretske:

A signal r carries the information that s is F = The conditional probability of s's being F, given r (and k), is 1 (but, given k alone, less than 1)

A set of relevant alternatives is used to determine the range of possibilities over which this probability assessment is made.

For reasons that are well established, a consequence of this account is that both the epistemic closure principle and the KK principle fail. According to various versions of the closure principle, if one knows that p and knows that p implies q then they know or are at least in a position to know that q [15]. Basically, this can fail in cases where the antecedent proposition and the consequent proposition of the known implication have different sets of relevant alternatives.

Dretske's classic zebra example [7, p. 1016] provides an amusing way to explain all of this. In this scenario, one is at a zoo where they go to the zebra section and see what they believe to be a zebra. Now, we can say that every time one receives a zebra visual signal in certain circumstances then that means there is a zebra before them and so the required probability is 1. Therefore this visual signal carries the information that there is a zebra (z) and any resulting belief that there is a zebra is knowledge. But for this judgement on the information-carrying status of the signal to be made, certain theoretically possible but non-actual irrelevant alternatives, such as ones in which the creature before the zoo goer is a cleverly disguised mule painted to resemble a zebra or a virtual zebra in a simulation, are not considered. Thus it is within the set of relevant alternatives, which say, consist of standard zoo scenarios, that a zebra visual signal carries the information that there is a zebra.

By receiving the information and forming the belief that the creature is a zebra the zoo goer comes to know that the creature is a zebra. Now it is a necessary truth that if something is a zebra then it is not a mule and we can rightly suppose that this is something the zoo goer knows. But given that 'not-mule' ($\neg m$) is judged against a more demanding set of relevant alternatives, a set that includes alternatives in which the creature is a disguised mule, the visual zebra signal does not provide sufficient information to rule out such alternatives and know that 'not-mule'. Thus to put it in terms used by Dretske, one can know that z (an ordinary proposition) and know the necessary implication $z \Rightarrow \neg m$ without knowing that $\neg m$ (a heavyweight or skeptical proposition). We:

> preserve ordinary knowledge by denying closure. We don't have to know the heavyweight implications of P to know P. I don't have to know I am not a deluded brain in a vat to see (hence, know) that there are cookies in the jar. I can, and often do, get information about P sufficient unto knowing that P is true without getting information about the heavyweight implications of P.[11, p. 23]

For related reasons, the KK principle is rejected. According to various versions of the KK (Knowing that One Knows) principle, if one knows some proposition p then they know or are at least in a position to know that they know that p [14]. As explained, one can see a zebra at the zoo and know that there is a zebra before them without being able to ascertain that the creature is not a painted mule, not a virtual zebra simulation, etc. Such irrelevant alternatives determine a set of conditions: the signal carries the information that 'zebra' given that 'painted mule' is not the case, 'virtual zebra' is not the case, etc. That these conditions obtain is not something the knower has to know though.

Similarly, a metal detector's beep carries the information that there is metal below given that the beep is not due to a flat battery, faulty circuit, etc. But whilst the beep's carriage of the information that there is metal relies on these conditions, the beep itself does not carry the information that none of these conditions have failed. Thus a signal can carry the information that p without carrying the information that it is carrying the information that p and an agent can know that there is metal without knowing that certain conditions required for this to be the case have obtained.

This framework has been used to reject the KK principle. As Dretske writes:

> modest contextualism (and, hence, externalism) provides an illuminating

> explanation of why KK fails. It fails because factual knowledge, according to modest contextualism, depends for its existence on circumstances of which the knower may be entirely ignorant. So the knower can know that P without knowing (as required by KK) that he knows that P. [10, p. 176]

So in the zebra-mule scenario for example one can know that the creature is a zebra without knowing that they know this because they do not know that required conditions such as not-mule have obtained.[1]

From here we can clearly see the relation between closure failure and KK principle failure. A knower can know some ordinary proposition p based on the information that p. This information relies on an irrelevant alternative q not obtaining and although $\neg q$ is a consequence of p, the knower does not have to know that $\neg q$. Rejection of the closure principle and the KK principle has made for substantial debate, with it being fair to say that the former has been more controversial.

2 Abominations

A common objection to accounts that deny closure is that they result in what DeRose termed 'abominable conjunctions' [6]. In general, these conjunctions are of the form $\mathrm{K}p \wedge \neg\mathrm{K}\neg q$, where p is an ordinary proposition and q is a heavyweight proposition implied by p, the following being an example:

> I know that I have hands but I do not know that I am not a handless brain in a vat

According to those like DeRose such statements are highly problematic and render such accounts of knowledge unacceptable. Although this might seem the case, we must be reminded that each of the conjuncts here is judged against different sets of relevant alternatives. Granted that such abominable conjunctions seem odd they are not contradictory and are a consequence of the same logic that results in closure denial.[2] As with closure, I think that the question of most importance to this issue concerns the legitimacy of shifting contexts (sets of relevant alternatives) across propositions in an argument [3]. Such conjunctions are something that Dretske is committed to though.

In a recent paper [17], Michael Veber argues that things are worse. He attempts to buttress the argument from abomination by adding another type of to the list, what he terms 'logical abominations'. As I see it though, a crucial consideration pertaining to such externalist accounts of knowledge is missed. As I will now argue, once this consideration is factored in,

[1]Okasha [16] argues that this type of standard externalist argument against the KK principle is fallacious. In [4] I argue that a better way to explicate failure of the KK principle along these externalist lines is to have the relevant alternatives for the proposition $\mathrm{K}z$ being greater and more demanding than the relevant alternatives for z, including as relevant certain alternatives that are judged irrelevant for z.

[2]In terms of epistemic logic, whilst $\mathrm{K}p \wedge \mathrm{K}(p \Rightarrow q) \wedge \neg\mathrm{K}q$ is a contradiction, the following similar representation in a multi-modal logic, where different accessibility relations correspond to different sets of relevant alternatives, is not: $\mathrm{K}_x p \wedge \mathrm{K}_y(p \Rightarrow q) \wedge \neg\mathrm{K}_z q$.

these 'logical abominations' are not something that Dretske is committed to. Veber lists 13 abominations but my point essentially undermines them all.

Consider the following skeptical argument:

1. I have hands
2. If I have hands, then I am not a brain in a vat
3. Therefore, I am not a brain in a vat

According to Veber,

> on Dretske's view, I know that the premises of this argument are true. And, although he denies various epistemic "closure" principle, Dretske is careful to distinguish those principle from modus ponens, which he does not deny. ... But according to Dretske, I do not know that the conclusion is true.[17, p. 1187]

Thus we get logical abomination number one:

A1: I know that argument is valid and that it has true premises but I don't know that its conclusion is true

According to my analysis, this charge of abomination fails because the knower in such a situation does not actually know that both of the premises are true. In particular, they do not know that 'hands' is true. This point essentially relies on an important distinction between knowing that p and knowing that *p is true*, with the latter requiring that the types of higher standards associated with heavyweight propositions and meta-knowledge be met. Thus knowing that p is true implies knowing that p but knowing that p does not imply knowing that p is true.

As has been covered, according to Dretske's account one can know an ordinary proposition such as 'zebra' without needing to rule out certain irrelevant alternatives such as 'not-mule'. As a result they can come to know that 'zebra' without knowing 'not-mule', despite the former entailing the latter. Also, they can know that 'zebra' without knowing that they know that 'zebra'. These externalist hallmarks suggest that knowing that p is different to knowing that p is true.

One can come to know that 'zebra' via a visual signal that provides them with sufficient information to rule out the relevant alternatives. But this does not suffice to know that it is true that there is a zebra. To meet the standards required to know the truth of some proposition, they must rule out a stronger set of relevant alternatives. Knowing that p requires that certain conditions, some of which the knower might be ignorant of, be met. But knowing that p is true requires knowing that these conditions have been met; to know the alethic status of a proposition p requires ruling out alternatives that are irrelevant to p in terms of knowing that p. Thus according to this analysis if one knows that p is true then they would be in a position to know the type of heavyweight propositions that p entails.

The position I espouse here leads to a point that can very well be associated with a theme of externalist accounts of knowledge such as Dretske's: the alethic status (of truth) of a proposition p does not need to be ascertained with certainty in order to know that p. As long as it is the case that p is true and that the obtainment of certain external conditions provides for a signal that carries the information that p, then an agent who forms a belief that p caused by such a signal can come to know that p.

This position also seems to fit in with two other aspects of Dretske's account. Firstly, his idea that such an externalist epistemology can accommodate the possibility that animals could know things without having to suppose that they are capable of sophisticated human epistemic faculties. In this case, they can know without possessing alethic concepts.

Secondly, Dretske's account focuses on perceptual knowledge *de re*. For example, if one sees the flag of Switzerland, without knowing that it is the flag of Switzerland, one can come to know *de re* that the flag of Switzerland has a cross without knowing it *de dicto*. In such a case, whilst the knower is in a position to make a claim like 'that flag has a cross' they are not in a position to make a claim like 'it is true that the flag of Switzerland has a cross'. Thus they can know *de re* that the flag of Switzerland has a cross without knowing that it is true that the flag of Switzerland has a cross.

It seems that this distinction between knowing that p and knowing that p is true has not been adopted in the literature. Even quotes from Dretske like the cookie jar one above and the following suggest that no distinction has been made:

> Closure tells us that when S knows that P is true and also knows that P implies Q, then not only must Q *be* true (*modus ponens* gets you this much), *S* must *know* it is true. [11, p. 13]

In fact from the case I have made here it follows that this stronger form of closure is valid. It is the following form that is not:

When S knows that P and also knows that P implies Q, *S* must *know* that Q

The general point to be taken from this is that because *modus ponens* is valid, knowing that some proposition is true means knowing that its consequences are true. Therefore, if a consequence is not known then neither is the antecedent proposition. More generally, at the very least, a schema for assessing epistemic arguments would be such that for every premise P an agent would know that P is true only if they have a conclusive reason for P relative to the set of relevant alternatives associated with the conclusion.

Applying all of this to the zebra-mule example, we can reason as follows that the truth of ‘zebra’ is not known:

(1) $\neg K \neg m$	(premiss)
(2) $K\neg m$ is true $\supset K \neg m$	(rule)
(3) $\neg K \neg m$ is true	(*modus tollens* on 1,2)
(4) $\neg K \neg m$ is true $\supset \neg Kz$ is true	(contraposition from strong closure)
(5) $\neg Kz$ is true	(*modus ponens* on 3,4)

Importantly, this conclusion does not rule out Kz. So whilst we evade logical abominations with this approach, we do get a conjunction that might be considered odd: $Kz \wedge \neg(Kz$ is true).

Thus though it is the case that in scenarios such as the standard zebra-mule one closure fails, closure of the stronger kind holds since ‘knows that zebra is true’ is a false premise and the logical abominations do not pose a problem. Alternative doxastic versions of the logical abominations such as the following do not pose a problem either given that belief is not affected by the same type of closure rejection as knowledge:

> I know that argument is valid and believe it has true premises but I don’t believe that its conclusion is true

In fact such a logical abomination makes it clearer that the real issue would be an agent who is committed to the premises but not committed to the conclusion despite there being no external factors such as varying relevant alternatives to differentiate the agent’s relationships to the premises and conclusion. Therefore assuming that agents do not disbelieve the consequences of their beliefs, in neither the epistemic or doxastic version is there a violation of adherence to logical reasoning.

As a segue to the next section I conclude this one by mentioning Veber’s eighth abomination:

A8: I know that the first disjunct is true. And I know that’s all it takes for a disjunction to be true. But for all I know, the disjunction is false.

This abomination is based on the claim that according to Dretske’s account knowledge is not necessarily closed under disjunction:

$$Kp \nvdash K(p \vee q)$$

Hawthorne, who as Veber mentions uses a different example to make a similar point, terms this principle Addition Closure. An example of this supposed failure is:

> I know that is a zebra
>
> but

I do not know (that is a zebra or that is not a mule)

As I will now discuss, a proper understanding of the logic of Dretske's account will show that he is not committed to the denial of this principle.

3 Principles of a Dretskean Epistemic Logic

What would a Dretskean epistemic logic look like? To begin with, it would not endorse the following principles:[3]

- $\mathrm{K}(p \supset q) \supset (\mathrm{K}p \supset \mathrm{K}q)$
- $\mathrm{K}p \supset \mathrm{KK}p$

and closure will be invalid:

- $\mathrm{K}p \wedge \mathrm{K}(p \supset q) \nvdash \mathrm{K}q$

Furthermore, results detailed in [5] show that closure under conjunction must also fail given Dretske's account:

- $\mathrm{K}p \wedge \mathrm{K}q \nvdash \mathrm{K}(p \wedge q)$

As an example of this, we can use the zebra-mule scenario to construct a case where a signal carries the information that A and the information that B without carrying the information that $A \wedge B$. Employing the following terminology

- $A \sqsupset B$ = A carries the information that B
- v = visual zebra signal
- z = zebra
- $\neg m$ = not-mule

consider the following two true statements of information carriage:

- $v \sqsupset z$

[3] $\supset$ is used to denote material implication and $\Rightarrow$ to denote strict or necessary implication.

- $v \sqsupset (z \wedge \neg m) \vee (m \wedge \neg z)$

If they were joined to get $v \sqsupset (z \wedge ((z \wedge \neg m) \vee (m \wedge \neg z)))$, then since $z \wedge ((z \wedge \neg m) \vee (m \wedge \neg z))$ is logically equivalent to $z \wedge \neg m$, this would lead to the unwanted $v \sqsupset \neg m$.[4] Therefore if the conjunction principle for information flow can fail to hold it follows that knowledge can also fail to close under conjunction. One could know that z and know that $(z \wedge \neg m) \vee (m \wedge \neg z)$ without being in a position to know their conjunction because they do not have the information that is their conjunction.

To add to this, denying the validity of conjunction closure means the following invalidity:

$$\mathrm{K}z \wedge \mathrm{K}(z \Rightarrow \neg m) \nvdash \mathrm{K}(z \wedge (z \Rightarrow \neg m))$$

We can interpret this by saying that if one knows that there is a zebra based on the visual information and that not-mule is a channel condition, then this piece of knowledge should not be 'mixed' with a knowledge statement that zebra entails not-mule. One is empirical knowledge that depends on a certain channel condition and the other is knowledge of a necessary conditional that involves the channel condition as a consequent.

Furthermore, we can also see that a principle of conjunction closure for knowledge does not sit well with certain combinations involving iterated knowledge. With regards to the standard zebra-mule scenario, it is fair to think that Dretske would assert the following; the first two are explicit in his account and the third presumably follows from the second:

1. $\mathrm{K}z$
2. $\neg\mathrm{K}\neg m$
3. $\mathrm{K}\neg\mathrm{K}\neg m$

If conjunction closure were to hold here then the resulting $\mathrm{K}(z \wedge \neg\mathrm{K}\neg m)$ is problematic; what is the set of relevant alternatives for the proposition $z \wedge \neg\mathrm{K}\neg m$ under which it could be known?

Continuing on, two principles that will be included in a Dretskean epistemic logic are:

- $\mathrm{K}(p \wedge q) \vdash \mathrm{K}p$ (Conjunction Distribution)
- $\mathrm{K}p \vdash \mathrm{K}(p \vee q)$ (Addition Closure)

These two principles are part of Dretske's idea of knowledge as a semi-penetrating operator [7] and contrary to claims (such as those soon to be discussed) that they do not hold given Dretske's account, I see no reason why they would fail.

[4] Given that:

1. if B and C are logically equivalent and $A \sqsupset B$ then $A \sqsupset C$
2. if $A \sqsupset (B \wedge C)$ then $A \sqsupset B$ and $A \sqsupset C$

Given these considerations, we can consider the system consisting of the following axioms and rules as suitable to capture the validities and invalidities of a Dretskean epistemic logic:

- Propositional Calculus (PC)
- The axiom: $\mathrm{K}p \supset p$
- The axiom: $\mathrm{K}(p \wedge q) \supset \mathrm{K}p$
- The rule: $\dfrac{p \equiv q}{\mathrm{K}p \equiv \mathrm{K}q}$ [where $p \equiv q$ is a theorem]

This system corresponds to a certain non-normal modal logic that can be give a neighborhood semantics [2]. The rule $\dfrac{p \supset q}{\mathrm{K}p \supset \mathrm{K}q}$ (Right Monotonicity) can be derived from this system and hence so can Addition Closure. The inappropriateness of closure conjunction is reinforced by the fact that its addition as an axiom to this system results in the validity of $\mathrm{K}p \wedge \mathrm{K}(p \supset q) \vdash \mathrm{K}q$.

Whilst Conjunction Distribution and Addition Closure seem like safe principles, some have argued otherwise. In one defence of closure Hawthorne [13, 12] argues that rejecting closure entails rejecting conjunction distribution. He starts off with the following:

The Equivalence Principle: If one knows *a priori* (with certainty) that P is equivalent to Q and knows P, and competently deduces Q from P (retaining one's knowledge that P), one knows Q

Hawthorne claims that Dretske's reasons for denying closure do not affect this principle and suggests that Dretske will accept it:

> [Dretske's] argument against closure relies on the following idea. Following recent usage, let us say that R [a conclusive reason] is "sensitive" to P just in case were P not the case, R would not be the case. Suppose one believes P on the basis of R, and that P entails Q. R may be sensitive to P and still not to Q. But notice that where P and Q are equivalent, there can be no such basis for claiming that while R can underwrite knowledge that P, it cannot underwrite knowledge that Q.[13, p. 31]

Conjunction Distribution is formulated as follows:

Distribution: If one knows the conjunction of P and Q, then as long as one is able to deduce P, one is in a position to know that P (and as long as one is able to deduce Q, one is a position to know that Q).[13, p. 31]

Using the following example, Hawthorne reasons that although distribution is extremely plausible, Dretske is committed to denying it.

> Suppose one knows that some glass g is full of wine on the basis of perception (coupled, perhaps, with various background beliefs). The proposition that g is full of wine is *a priori* equivalent to the proposition
>
> g is full of wine and ∼g is full of non-wine that is colored like wine
>
> So by equivalence one knows that conjunction. Supposing distribution, one is in a position to know that
>
> ∼g is full of non-wine that is colored like wine

As I will now show, Hawthorne's argument does not provide grounds for the claim that Dretske is committed to denying distribution. To begin with, Hawthorne's wine example has the same structure as Dretske's original zebra example that we have been working with, where wine corresponds to zebra and non-wine corresponds to mule. So we will return to the zebra-mule example.

In [1] the authors argue, in concordance with my own formal analysis, that Dretske's account actually entails giving up the equivalence principle instead of conjunction distribution. According to them, in cases like the zebra-mule scenario Dretske's reasons for denying closure do have force against the equivalence principle. Let:

- p = a certain animal x is a zebra
- q = x is not a painted mule
- R = x appears to be a zebra (R is a conclusive reason)

As they write:

> Suppose (i) R is sensitive to p, that is, if it were not the case that x is a zebra, x would not appear to be a zebra; (ii) S knows p on the basis of R; (iii) S knows *a priori* that p if and only if $p\&q$; (iv) S competently deduces $p\&q$ from p; and (v) if x were a painted mule, x would appear to be a zebra. Although the Equivalence Principle implies that S knows $p\&q$, the following considerations show that Dretske's analysis of knowledge implies that S doesn't know $p\&q$ because R is insensitive to $p\&q$. In virtue of (i) and (v), it follows that if it were not the case that x is a zebra, then x would not be a painted mule (for otherwise x would appear to be a zebra). But if it were not the case that x is both a zebra and not a painted mule, i.e., if it were the case that x is either a non-zebra or a painted mule, then x might be a painted mule, in which case x would appear to be a zebra. Consequently, R is insensitive to $p\&q$. That is, if it were not that case that x is a zebra and not a painted mule, x might appear to be a zebra. Thus, even though the Equivalence Principle implies that S knows that $p\&q$, contrary to Hawthorne's contention, Dretske's reasons for denying closure *do have force* against the Equivalence Principle. [1]

So once we get our heads around the unorthodox logic that accompanies Dretske's denial of closure we can see that his reasons for denying closure do have force against the Equivalence Principle. In informational terms, the point being made here is that whilst z and $z \wedge \neg m$ are (truth) equivalent propositions, they are judged relative to different sets of relevant alternatives and are not equivalent with regards to their informational status. So whilst $v \sqsupset z$, it is not the case that $v \sqsupset (z \wedge \neg m)$ and hence it is not the case that $(\neg z \vee m) \sqsupset \neg v$. Thus one can know z without knowing $z \wedge \neg m$, despite their necessary equivalence.

The non-normal modal system outlined above gets the right validities/invalidities and technically deals with the issue Hawthorne raises. It both validates the Distribution Principle and invalidates the Equivalence Principle. The Distribution Principle is validated given that it is an axiom. The Equivalence Principle however is not valid: $\mathrm{K}p \wedge \mathrm{K}(p \Leftrightarrow q) \nvdash \mathrm{K}q$. In terms of our example, it is not the case that z is logically equivalent to $z \wedge \neg m$. So whilst $\mathrm{K}(z \Leftrightarrow (z \wedge \neg m))$ is true, since z and $(z \wedge \neg m)$ are not logically equivalent, knowledge of the former does not entail knowledge of the latter. Rather, it is $z \wedge (z \Rightarrow \neg m)$ which is logically equivalent to $z \wedge \neg m$. So by logic we have $\mathrm{K}((z \wedge (z \Rightarrow \neg m)) \equiv (z \wedge \neg m))$ to begin with and it doesn't boil down to whether or not z is known but rather whether or not $z \wedge (z \Rightarrow \neg m)$ is known. But as we have seen, given the rejection of conjunction closure, although we have $\mathrm{K}z$ and $\mathrm{K}(z \Rightarrow \neg m)$ it does not follow that $\mathrm{K}(z \wedge (z \Rightarrow \neg m))$.

Also, it is worth noting that $\mathrm{K}(z \Leftrightarrow (z \wedge \neg m))$ breaks down into $\mathrm{K}(z \Rightarrow (z \wedge \neg m))$ and $\mathrm{K}((z \wedge \neg m) \Rightarrow z)$ anyway, so it seems that Hawthorne has implicitly performed an illegitimate closure operation in arguing for closure. Formally, this can be seen in the following. The conditionals involved in the closure principle and the equivalence principle are necessary conditionals, thus formalised in terms of modal logic the equivalence principle is:

$$\mathrm{K}(\Box(p \equiv q)) \wedge \mathrm{K}p \vdash \mathrm{K}q$$

(1) $\mathrm{K}z$	(premise)
(2) $\mathrm{K}\Box(z \equiv (z \wedge \neg m))$	(premise)
(3) $\mathrm{K}(\Box(z \supset (z \wedge \neg m)) \wedge \Box((z \wedge \neg m) \supset z))$	(from 2)
(4) $\mathrm{K}\Box(z \supset (z \wedge \neg m))$	(Conjunction Distribution on 3)
(5) $\Box(z \supset (z \wedge \neg m)) \supset (z \supset (z \wedge \neg m))$	(Modal T axiom)
(6) $\mathrm{K}\Box(z \supset (z \wedge \neg m)) \supset \mathrm{K}(z \supset (z \wedge \neg m))$	(Right Monotonicity on 5)
(7) $\mathrm{K}(z \supset (z \wedge \neg m))$	(*Modus Ponens* on 5,6)

It is at this point that an illegitimate closure operation would need to be performed to get $\mathrm{K}(z \wedge \neg m)$.

As Veber alludes to, Hawthorne also argues that Dretske is committed to denying Addition Closure [12, p. 39]. The argument provided seems less clear than the one provided against Conjunction Distribution, but my interpretation of it gives the following formal representation:

It is at this point that an illegitimate closure operation would need to be performed to get $\mathrm{K}\neg m$.

Quite simply, if $\mathrm{K}\neg m$ does not follow from $\mathrm{K}z \wedge \mathrm{K}(z \Rightarrow \neg m)$, then $\mathrm{K}\neg m$ does not follow from $\mathrm{K}(z \vee \neg m) \wedge \mathrm{K}((z \vee \neg m) \Rightarrow \neg m)$; adding the consequent of the known implication as a disjunct to the antecedent of the known implication will not give closure in such cases.

(1) Kz	(premise)
(2) K$(z \vee \neg m)$	(Right Monotonicity on 1)
(3) K$\Box((z \vee \neg m) \equiv \neg m)$	(premise)
(4) $\Box((z \vee \neg m) \equiv \neg m) \supset ((z \vee \neg m) \equiv \neg m)$	(Modal T axiom)
(5) K$\Box((z \vee \neg m) \equiv \neg m) \supset$ K$((z \vee \neg m) \equiv \neg m)$	(Right Monotonicity on 4)
(6) K$((z \vee \neg m) \equiv \neg m)$	(*Modus Ponens* 3,5)
(7) K$((z \vee \neg m) \supset \neg m)$	(From 6)

4 Conclusion

The main point of this paper has been to introduce the idea that an externalist epistemology such as Dretske's leads to a distinction between knowing that p and knowing that p is true. This means that although for 'knows that' the principle of epistemic closure is rejected and abominable conjunctions arise, 'knows that is true' does not face such issues. The grounds for this distinction remain to be further developed and its consequences further explored. We have also seen that once the non-normal nature of a Dretskean epistemic logic is appreciated, closure denial does not mean sacrificing conjunction distribution and addition closure so the costs of denying closure are not as great as some have made out.

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